

Lattice Field Theory and its Applications

Physics 578B, Spring 2023

Time & Room: TBD

Instructor: Steve Sharpe

Prerequisites: 570 & 571 (first two quarters of QFT) or equivalent

This special topics course is aimed particularly at students working in or interested in particle and nuclear theory; it should also be accessible to any interested student who has a basic grounding in Quantum Field Theory.

The focus of the course will be to explain how lattice regularization allows one to study fundamental issues in a non-perturbative way. Examples include the “triviality” of certain field theories, the appearance of non-perturbative phenomena such as confinement and chiral-symmetry breaking, and the non-perturbative meaning of topological charge and index theorems. In addition, some time will be spent on “applied QFT” issues such as how one can determine scattering and decay amplitudes in field theories from calculations of finite-volume Euclidean correlation functions.

The aim is that a student completing this course should know what can, and what cannot, be done using lattice regularization, and have some idea about how calculations are performed and how theoretical issues are tackled. This knowledge should be useful to all particle and nuclear theorists. Although the main focus will be on theoretical lattice calculations, some mention of the status of present numerical calculations will be made.

This course is largely orthogonal and complementary to PHYS 578A : Quantum Information and Simulation for Scientific Applications, being taught by Prof. Martin Savage.

Topics to be covered will be drawn from the following list:

- Theory of scalar fields on the lattice. Transfer matrix. Reflection positivity. Naive continuum limit.
- Relation of lattice scalar field theory to lattice spin models. Renormalization group and universality. Expansions: weak coupling and “hopping parameter”.
- Lüscher-Weisz demonstration of triviality of scalar field theories.
- Gauge fields on the lattice. Wilson and Polyakov loops. Strong coupling expansions and demonstration of confinement at strong coupling.

Abelian vs non-abelian theories: deconfinement transition in lattice QED.

- Transfer matrix for pure gauge theory. Continuum limit and relation of lattice and continuum perturbation theory. Systematically understanding discretization effects using Symanzik effective Lagrangian, and methods for reducing such effects by “improving” the gauge action.
- Brief discussion of simulation methods and calculation of glueball spectrum. Lattice symmetries and implications for spectrum.
- Lattice fermions. Basic formulation, “doubling” problem and Wilson fermions. Brief mention of staggered fermions. Demonstration of spontaneous chiral symmetry breaking in strong coupling. Shortcomings of Wilson and staggered fermions.
- Lattice fermions with exact chiral symmetry: Domain-wall and overlap fermions. Index theorem. Using “Wilson-flow” to define topology.
- Finite volume methods. How to obtain scattering and decay amplitudes from the finite-volume spectrum.
- Lattice QCD and related theories at finite density.
- Large- N_c volume reduction on the lattice.

This is a C/NC class. The tentative plan is for 5 homework sets, with the possibility of replacing one or more of these with a project if desired.