- I. [45 points total] Lecture multiple-choice questions
 - 1. [5 points] A mass attached to a spring executes simple harmonic motion with angular frequency 4 radians per second. At time t = 0 the displacement is x = 1.5 m, and the velocity is 10.4 m/s. What is the speed of the mass when it returns to its equilibrium position x = 0.0 m?
 - A. 2.6 m/s
 - B. 0.0 m/s
 - C. 12.0 m/s
 - D. 20.8 m/s
 - E. Not enough information is given.

- 2. [5 points] A mass attached to a spring passes the equilibrium position x = 0.0 m at t = 0.0 s with a certain positive velocity. For times between t = 0.0 s and t = 1.0 s, it keeps moving in the positive x direction. At t = 1.0 s its kinetic energy is half of what it was at t = 0.0 s. What is the period of the motion?
 - A. 3.0 s
 - B. 2.0 s
 - C. 4.0 s
 - D. 8.0 s
 - E. 16.0 s

- 3. [5 points] A particle in one dimension experiences a force $F(x) = -x(x^2 1)$. How many equilibrium points are there, and how many of those are stable?
 - A. 2 equilibrium points, 2 of them stable
 - B. 3 equilibrium points, 2 of them stable
 - C. 3 equilibrium points, 3 of them stable
 - D. 3 equilibrium points, 1 of them stable
 - E. 3 equilibrium points, none of them stable

- 4. [5 points] A particle undergoing simple harmonic motion at a certain angular frequency and amplitude experiences a maximum acceleration of 30 m/s^2 . Now let's double both the amplitude and the angular frequency. What is the new maximum acceleration?
 - A. 15 m/s^2
 - B. 30 m/s^2
 - C. 60 m/s^2
 - D. 120 m/s^2
 - E. 240 m/s^2
- 5. [5 points] There is a splice from a string of mass density μ_1 on the left to another string of mass density μ_2 on the right. You notice that if you send in a pulse from the left, the reflected pulse is inverted. Which of the following changes can you make to the system so that, for a pulse sent in from the left, the reflected pulse is not inverted?
 - A. increase μ_2
 - B. increase the tension in the string
 - C. decrease the tension in the string
 - D. decrease the width of the incoming pulse
 - E. none of the above
- 6. [5 points] At instant t_1 a circular ripple has wavefront radius R_1 and energy in the pulse E_1 . At a later instant t_2 the radius of the wavefront doubled $R_2 = 2R_1$. Assume there is no energy dissipation. How does the energy per unit length along the wavefront at instant t_2 compare with that at instant t_1 ?
 - A. The energy per unit length at instant t_2 equals that at instant t_1 .
 - B. The energy per unit length at instant t_2 is $1/\sqrt{2}$ that at instant t_1 .
 - C. The energy per unit length at instant t_2 is half that at instant t_1 .
 - D. The energy per unit length at instant t_2 is one quarter that at instant t_1 .
 - E. The energy per unit length at instant t_2 is one eighth that at instant t_1 .

- 7. [5 points] Two point-sources in a pond, located at (x, y) coordinates of (0 m, 0 m) and (0 m, 5 m), respectively, are oscillating in phase. The point (12 m, 0 m) is on an antinodal line. Which of the following is **not** a possible value for the wavelength?
 - A. ⅓m
 - B. ⅔ m
 - C. 1 m
 - D. ¼m
 - E. ⅓ m
- 8. [5 points] Assume the same geometry as problem 7, with again the point sources oscillating in phase. However, now the point (12 m, 0 m) is on a **nodal** line. What is the **largest** value of the wavelength consistent with this scenario?
 - A. ⅔ m
 - B. 1 m
 - C. 2 m
 - D. 3 m
 - E. 4 m
- 9. [5 points] An ambulance and a car are approaching each other, each traveling with the same speed. The ambulance's siren's frequency is 800 Hz, but the observer in the car hears a frequency of 820 Hz. Now consider a situation where the same ambulance and car are driving away from each other, both with the same speed as before. What is the frequency heard by the driver of the car in this new situation?
 - A. 760.5 Hz
 - B. 770.5 Hz
 - C. 780.5 Hz
 - D. 790.5 Hz
 - E. 800.5 Hz

- II. [15 points total] Lab multiple-choice questions
 - 10. [5 points] In lab A1, a lab team produces a standing wave with two antinodes using a string with a fixed length under a fixed tension. Each team member determines at which frequency this occurs and notes the value shown on the display of the function generator as follows: 37.54 Hz, 36.47 Hz, 38.13 Hz, and 37.81 Hz.

Which of the following values is consistent with the rules that we use on how to report the best estimate of this frequency?

- A. 37.49 ± 0.01 Hz
- B. 37.5 ± 0.7 Hz
- C. $37 \pm 1 \text{ Hz}$
- D. 37.49 ± 0.7 Hz
- E. $37.5 \pm 1 \text{ Hz}$

- 11. [5 points] In Lab A1, a lab team measured the **frequency** of standing waves with two antinodes as they varied **the mass of the hanging weight**. They kept all other aspects of the set-up constant. In lab A2, the team is tasked to evaluate whether the simulation of balls connected by springs reproduces the behavior of standing waves on the string they observed in Lab A1. What should be their dependent and independent variables?
 - A. Frequency as dependent variable and mass of the balls as independent variable
 - B. **Frequency** as dependent variable and **number of the balls** as independent variable
 - C. Frequency as dependent variable and spring constant as independent variable
 - D. Number of antinodes as dependent variable and number of the balls as independent variable
 - E. Number of antinodes as dependent variable and mass of the balls as independent variable

- 12. [5 points] Suppose that you are testing a model that relates quantities A and B as follows: $A = \frac{k}{\sqrt{B}}$, where k is a constant. You measure the values of A as you vary B. What is the best way to produce a linearized plot of your data to test the model?
 - A. *A* on the vertical axis, and *B* on the horizontal axis
 - B. \sqrt{A} on the vertical axis, and *B* on the horizontal axis
 - C. *A* on the vertical axis, and $\frac{1}{\sqrt{B}}$ on the horizontal axis
 - D. *A* on the vertical axis, and \sqrt{B} on the horizontal axis
 - E. $\frac{1}{\sqrt{A}}$ on the vertical axis, and *B* on the horizontal axis

III.[25 points total] Lecture free response questionsYou must show your work to get the full credit.

Consider the following scenario for the next three questions.

13. [5 points] A string of length L with wave-speed c is tied to a pole at its right endpoint, and moved up and down at its left endpoint, in such a way that a standing wave is formed with the left endpoint an anti-node. Write down a formula, in terms of L and c, for the lowest frequency f_1 at which this is possible, and sketch the corresponding standing wave.

Solution: For the lowest frequency (i.e. fundamental frequency), lambda = 4L. Since $c=f^*$ lambda, we have f=c/(4L).

- [1] point for realizing lambda = 4L
- [2] points for using c=f*lambda to solve f=c/(4L)
- [2] points for correct sketch

14. [7 points] Consider the same scenario as in problem 13, but now imagine decreasing the tension in the string (assume the mass per unit length stays constant). Assuming the string is driven at the frequency f_1 you found above, by how much do you have to decrease the tension in order to form a standing wave with two anti-nodes (including the one at the left endpoint)?

Solution: For the new configuration, $L = (\frac{3}{4})$ lambda. So $c/(4L) = 3c_new / (4L)$, so $c_new = \frac{3}{2}c$. This means that the tension has to be decreased by a factor of $3^2 = 9$.

[3] points for figuring out that lambda becomes 4/3 L now

[2] points for using c = f*lambda and realizing that changing lambda while keeping f fixed means c changes

[2] points for using the formula for c in terms of tension to see that tension changes by a factor of 9.

15. [3 points] How much more power is delivered to the string in problem 13 as compared to problem 14, assuming the same amplitude in each case? That is, find the ratio. Solution: the formula for power is proportional to the mass density per unit length, amplitude squared, omega squared, and c. The only thing that changes is c. It changes by a factor of 3, so the answer is 3

[1] point for correct formula for power

- [1] point for realizing that only c changes
- [1] point for calculation (don't take off if mistake from problem 14 is propagated).

Use the following scenario for the next two questions.

A helicopter is hovering directly overhead. It's emitting 12.56 Watts of power as sound uniformly in all directions. The measured intensity / on the ground directly below the helicopter is 10^{-4} W / m².

16. [6 points] What is the intensity level (in decibels) that an observer on the ground hears, and what is the altitude of the helicopter (in meters)?
Solution: 4 pi (d)^2 * (10^{-4} W / m^2) = 12.56 W, so d = 100 m. The intensity level is

 $(10 \text{ dB}) * \log(I / I_0) = 80 \text{ dB}$, where $I_0 = 10^{-12} W / m^2$.

- [2] points for intensity level. (1 point for correct formula, one for correct answer)
- [2] points for correct formula relating distance and intensity
- [2] points for using that formula to correctly get the distance

17. [4 points] An eardrum of an observer on the ground directly below the helicopter absorbs about 10⁻⁸ Joules of this energy in one second when the eardrum is facing the helicopter directly. What is the area of the eardrum?

Solution: The power absorbed by the ear, P_ear, is $10^{-8} Joules / s = 10^{-8} W$. We have P_ear = A_ear * $10^{-4} W / m^2 = 10^{-8} W$. So A_ear = $10^{-4} m^2 = 1 cm^2$. [1] point for knowing that J / s = W

- [2] points for setting up equation relating area, intensity, and power.
- [1] point for correct numerical answer

IV. [15 points total] Tutorial free response questions

Use the following scenario for the next three questions.

A pulse is traveling at 1 m/s toward the end of a spring, which is **fixed** to a wall. The shape of the spring at t = 0 s is shown at right. Note that the side length of each square in the figure is 10 cm.

18. [8 points] On the diagram at right, draw the shape of the spring at t = 0.4 s. <u>Make sure to include and</u> <u>label all pulses</u> you used to determine your answer. The shape of the pulse at t = 0 s is shown in gray for your reference.





[1 point] correct incident pulse shape

[1 point] correct incident pulse horizontal position

[2 points] correct virtual pulse orientation

[1 point] correct virtual pulse horizontal position

[1 point] attempting to apply superposition

[2 points] correctly applying superposition (consistent with their incident and virtual pulses)



19. [7 points] At what time and what position (measured from the wall) on the string does the maximum possible displacement of the string occur? Explain your reasoning.

At t = 0.45 s, the peak labeled "A" has moved 2 squared to the left, reflected (and inverted), and moved 2.5 squared



to the right. Also, the peak labeled "B" has moved to 4.5 squared to the left. These peaks are at the same location and constructively interfere.

- [2 points] t = 0.45 s
- [1 point] The idea that two peaks add.
- [2 points] The idea that A and B are at the same location.
- [2 points] 25 cm from the wall

