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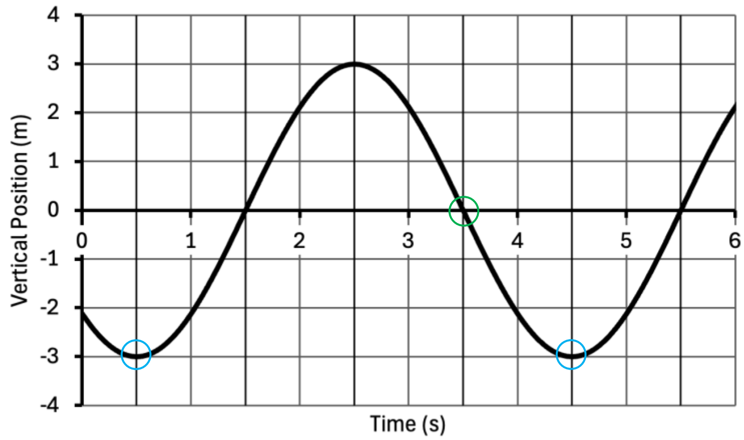
1. [3 pts] The figure at right shows the position-versus-time graph for an object in SHM. For the time interval shown in the graph, (a) at what time(s) does the particle move with its maximum negative velocity and (b) at what time(s) is the particle moving at its maximum positive acceleration?



A) (a) 1.5 s, 5.5 s (b) 2.5 s

B) (a) 1.5 s, 5.5 s (b) 0.5 s, 4.5 s

C) (a) 3.5 s (b) 2.5 s

D) (a) 3.5 s (b) 1.5 s, 5.5 s

E) (a) 3.5 s (b) 0.5 s, 4.5 s

The object is moving at its maximum negative velocity when it is moving through the equilibrium position from a positive position to a negative position. This occurs at t = 3.5 s. We can also look for the times when the slope of the position-time graph has its maximum negative value, which is at t = 3.5 s (green circle).

The object has its maximum positive acceleration when the object is at its maximum negative position (at t = 0.5 s and t = 4.5 s, blue circles). The restoring force at this location is at a maximum value and points in the positive direction. By Newton's second law, the largest net force corresponds to the largest acceleration.

2. [4 pts] Which of the equations below best describes the position-time function for the object in the previous question? The vertical axis intercept coordinate is (0 s, -2.12 m).

A) $y(t) = (3\text{ m})\cos\left(\left(\frac{\pi}{2}\frac{\text{rad}}{\text{s}}\right)t + 2.36\text{ rad}\right)$

B) $y(t) = (3\text{ m})\cos\left(\left(\frac{\pi}{2}\frac{\text{rad}}{\text{s}}\right)t - 2.36\text{ rad}\right)$

C) $y(t) = (3\text{ m})\cos\left(\left(\frac{\pi}{4}\frac{\text{rad}}{\text{s}}\right)t - 2.36\text{ rad}\right)$

D) $y(t) = (6\text{ m})\cos\left(\left(\frac{\pi}{4}\frac{\text{rad}}{\text{s}}\right)t - 0.784\text{ rad}\right)$

E) $y(t) = (3\text{ m})\cos\left(\left(\frac{\pi}{2}\frac{\text{rad}}{\text{s}}\right)t + 0.784\text{ rad}\right)$

We know that the initial position is (0 s, -2.12 m), the period is 4 s and the amplitude is 3.0 m. We can use this information to solve for the phase constant.

$$y(t) = A\cos(\omega t \pm \phi) = A\cos\left(\frac{2\pi}{T}t \pm \phi\right)$$

$$y(0) = -2.12\text{ m} = (3\text{ m})\cos(\omega(0) \pm \phi)$$

$$y(0) = -2.12\text{ m} = (3\text{ m})\cos(\phi)$$

$$\cos(\phi) = \frac{-2.12\text{ m}}{3\text{ m}}$$

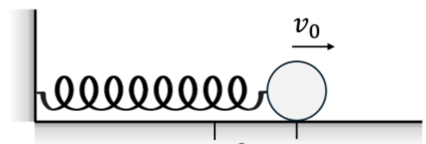
$$\phi = \cos^{-1}\left(-\frac{2.12\text{ m}}{3\text{ m}}\right) = \pm 2.36\text{ rad}$$

Since the object is moving in the negative direction at t = 0, the phase constant is positive.

We can now write the wave equation:

$$y(t) = (3\text{ m})\sin\left(\frac{2\pi}{4}\frac{\text{rad}}{\text{s}}t + 2.36\text{ rad}\right) = (3\text{ m})\sin\left(\left(\frac{\pi}{2}\frac{\text{rad}}{\text{s}}\right)t + 2.36\text{ rad}\right)$$

3. [3 pts] A ball of mass m_0 that is attached to an ideal spring with spring constant k_0 oscillates horizontally on a frictionless table. The ball is located at *x* = 0 cm when the spring is in its equilibrium position. The ball's velocity is v_0 when $x = x_p$. What is the amplitude of oscillation?



A) $A = \sqrt{\frac{m_0 v_0^2}{k_0} + x_p^2}$

B) $A = \sqrt{m_0 v_0^2 + k_0 x_p^2}$

C) $A = m_0 v_0^2 + \sqrt{k_0 x_p}$

D) $A = \sqrt{m_0 v_0^2} + x_p$

E) Other

The total energy of a SHM oscillator can be written as:

$$E_{total} = U_{max} = K_{max} = U + K$$

$$U_{max} = \frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

For this question, when the object is at $x = x_p$, it moves with a speed $v = v_0$.

$$\frac{1}{2}kA^2 = \frac{1}{2}kx_p^2 + \frac{1}{2}mv_0^2 \Longrightarrow A^2 = x_p^2 + \frac{mv_0^2}{k} \Longrightarrow A = \sqrt{x_p^2 + \frac{mv_0^2}{k}}$$

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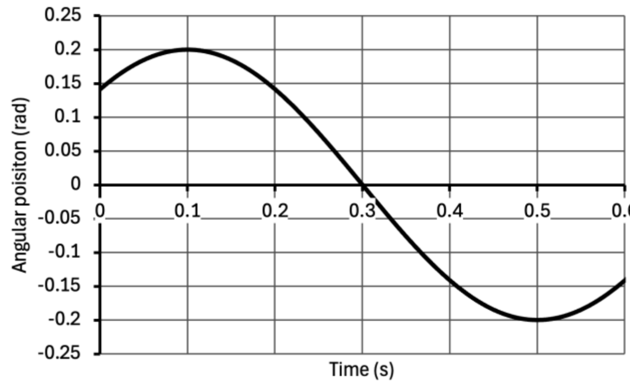
4. [3 pts] The angular position-versus-time curve of a simple pendulum is shown at right. What is the length of the pendulum?



A) 3.9 cm

B) 8.9 cm

C) 16 cm

D) 36 cm

E) 1.0 m

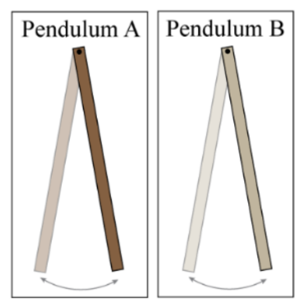
The period of a simple pendulum is given as:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

From the graph, we can see that the half the period (time from crest to trough) is 0.4 seconds. This means that the period is 0.8 seconds. We can now solve for the length.

$$l = \frac{T^2 g}{4\pi^2} = 0.16 \text{ m} = 16 \text{ cm}$$

5. [4 pts] A physical pendulum, pendulum A, is formed from a uniform rod of mass $2m$ and length L. It is allowed to pivot about one of its ends. $\left(I_{rod} = \frac{1}{12}mL^2\right)$



A second physical pendulum, B, is formed from a uniform rod also of length L, but of mass m. It is also allowed to pivot from one of its ends.

Is the period of pendulum A *greater than, less than,* or *equal to* the period of pendulum B?

A) Greater than

B) Less than

C) Equal to

D) More information is needed.

The period of a physical pendulum is given as:

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

The moment of inertia term includes a mass term, so the period is independent of the mass. The pendula also have the same length, so they will have the same period.

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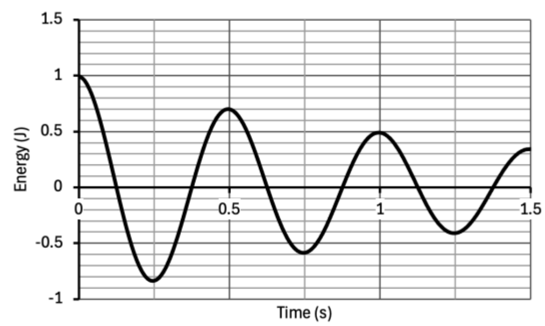
More detailed solution

A rod that pivots about one of its ends has a moment of inertia equal to $\frac{1}{3}mL^2$*. Since the rods are uniform, the value of d will be equal to L/2 (distance between pivot and the center of mass). We can write the period as follows:*

$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{1/3mL^2}{mgL/2}} = 2\pi\sqrt{\frac{2}{3}\frac{L}{g}}$$

Now we can see that the period of these pendula is only a function of their length. Since both pendula have the same length, they also have the same period.

6. [4 pts] The mechanical energy of a damped oscillator is shown as a function of time at right. What is the value of the time constant (τ) for this damped oscillator?



A) 0.25 s

B) 0.53 s

C) 0.82 s

D) 1.1 s

E) 1.4 s

The energy of a damped oscillator is given by the following equation:

$$E(t) = E_0 e^{-t/\tau}$$

We can solve for the time constant as follows:

$$\frac{E(t)}{E_0} = e^{-t/\tau}$$

$$\ln\frac{E(t)}{E_0} = \ln e^{-t/\tau} = \frac{-t}{\tau}$$

$$\tau = \frac{-t}{\ln\frac{E(t)}{E_0}}$$

At t = 0.5 s, the energy is 0.7 J and the initial energy is 1.0 J. We can use this information to solve for the time constant.

$$\tau = \frac{-0.5\text{ s}}{\ln\frac{0.7\text{ J}}{1.0\text{ J}}} = 2.2\text{ s}$$

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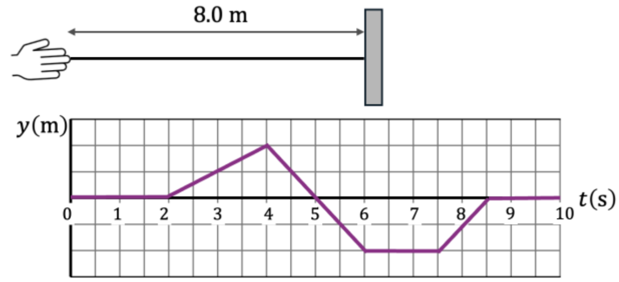
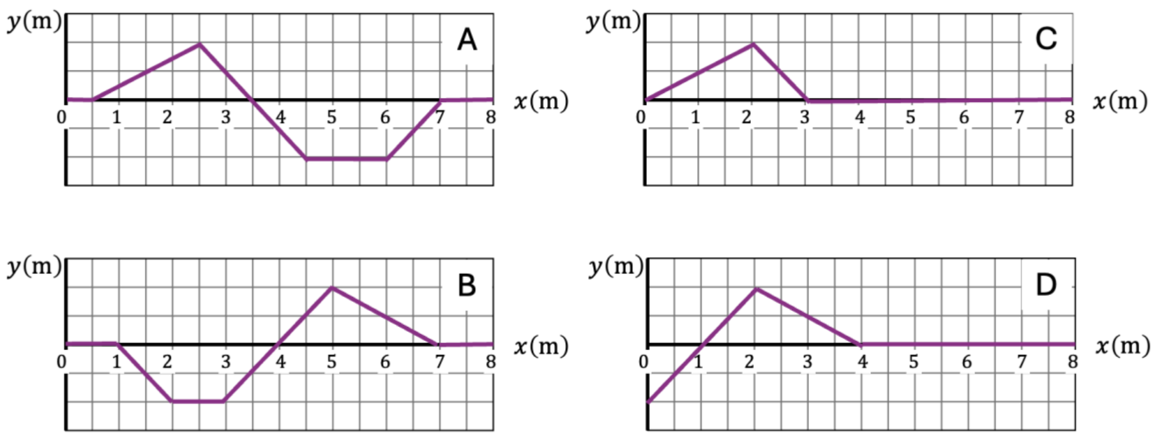
7. [4 pts] A student creates a pulse in a 8-m long string at $t = 0$ s. The pulse moves at 1 m/s. The figure at right shows a displacement curve (history graph) of a point on the string that is located 2 m from the student's hand for a time interval $t = 0$ to $t = 10$ s.



Which of the figures below could correctly represent a wave function (snapshot graph) of the pulse created by the student?



A) Figure A only

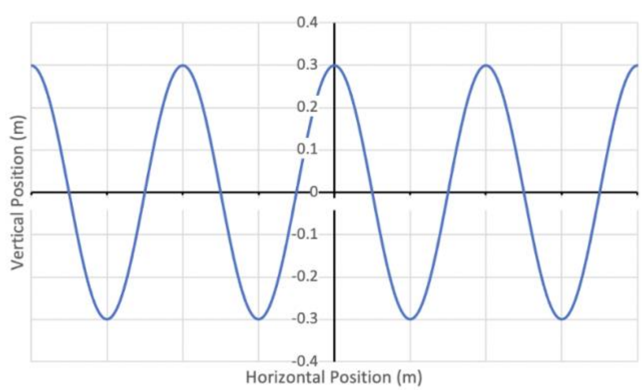
B) Figure A and B

C) Figure A and C

D) Figure B and D

E) Figure B only

8. [4 pts] A sinusoidal wave in the ocean has an amplitude of 0.300 m and a period of 8.33 s. The wave moves in the negative x-direction with a speed of 1.20 m/s. A snapshot graph of the wave is shown below at $t = 0$ s. What is the vertical position of the surface of the wave at $x = 4.00$ m at $t = 2.40$ s?



A) +0.20 m

B) +0.14 m

C) +0.062 m

D) -0.11 m

E) -0.15 m

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The traveling wave equation tells us the vertical position of a traveling wave at a given position x at a particular instant t. Since the pulse is at a maximum positive displacement at x = 0 m, I will write the equation as a cosine function rather than a sine function with a phase shift.

$$D_y(x,t) = A\cos\left(\frac{2\pi}{\lambda}x \pm \frac{2\pi}{T}t\right)$$

The wavelength can be written as the product of the speed and the period ($vT = \lambda$), and since the wave is moving in the negative x-direction, we use a positive sign between the two terms.

$$D_y(x,t) = A\cos\left(\frac{2\pi}{vT}x + \frac{2\pi}{T}t\right) = A\cos\left(\left(\frac{2\pi}{T}\right)\left(\frac{x}{v} + t\right)\right)$$

$$D_y(x,t) = (0.3\text{ m})\cos\left(\left(\frac{2\pi}{8.33\text{ s}}\right)\left(\frac{4\text{ m}}{1.20\text{ m/s}} + 2.4\text{ s}\right)\right) = -0.11\text{ m}$$

9. [4 pts] A journalist for the The Daily attends the Guns n Roses concert at the Gorge. The journalist gets access to the pit area and stands 30.0 m from the speakers on the right of the stage. If the sound intensity level at the journalist's location is 88.0 dB, what is the sound intensity at her location?

A) $2.59 \times 10^{-3}\text{ W/m}^2$

B) $3.16 \times 10^{-3}\text{ W/m}^2$

C) $6.31 \times 10^{-4}\text{ W/m}^2$

D) $3.84 \times 10^{-4}\text{ W/m}^2$

E) $1.58 \times 10^{-3}\text{ W/m}^2$

The relationship between intensity and sound intensity level is given by the following relationship.

$$\beta = 10\text{ dB}\log\frac{I}{I_0}$$

We can write the equation in terms of I as follows:

$$I = 10^{\beta/10}(I_0)$$

$$I = 10^{88\text{ dB}/10}(1 \times 10^{-12}\text{ W/m}^2) = 6.31 \times 10^{-4}\text{ W/m}^2$$

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10. [4 pts] A ambulance siren emits a sound of frequency 440.0 Hz. A student perceives a frequency of 422.4 Hz. The student also perceives that the speed of the sound waves has increased. Which of the following is true? Note that v_{sound} represents the speed of sound in air.

A) The ambulance is stationary and the student is moving at $14.4\ \text{m/s}$ toward the ambulance.

B) The ambulance is stationary and the student is moving at $13.7\ \text{m/s}$ away from the ambulance.

C) The ambulance is stationary and the student is moving at $14.4\ \text{m/s}$ away from the ambulance.

D) The student is stationary and the ambulance is moving at $13.7\ \text{m/s}$ away from the student.

E) The student is stationary and the ambulance is moving at $14.4\ \text{m/s}$ toward from the student.

Since the change in frequency is due to a perceived change in the wave speed, the student must be moving (the observer). And since the perceived frequency is lower than the source frequency, the student must be moving away from the stationary ambulance.

For a stationary source, the Doppler effect equation simplifies to:

$$f_o = \left(1 - \frac{v_0}{c}\right) f_s$$

$$\frac{v_0}{c} = 1 - \frac{f_o}{f_s}$$

$$v_0 = \left(1 - \frac{f_o}{f_s}\right) c = \left(1 - \frac{422.4\ \text{Hz}}{440\ \text{Hz}}\right) 343\,\frac{\text{m}}{\text{s}} = 13.7\ \text{m/s}$$

11. [3 pts] A 66.0 cm string is tied down at both of its end, and the tension in the strong is 15.6 N. The linear mass density of the string is measured to be 0.00863 kg/m and the string forms a standing wave that oscillates at 193.0 Hz. How many antinodes does this standing wave have?

A) 1

B) 2

C) 3

D) 4

E) 6

The frequency of a standing wave on a string that is tied at both ends is given as follows:

$$f_m = m\frac{v}{\lambda} = m\frac{\sqrt{\frac{T}{\mu}}}{2l}$$

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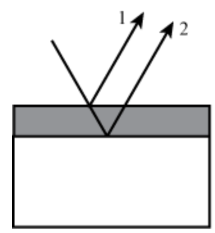
We can arrange this equation for m (the mode number, which also equals the number of antinodes) as follows:

$$m = \frac{2 f_m l}{\sqrt{\frac{T}{\mu}}}$$

$$m = \frac{2(193\text{ Hz})(0.66\text{ m})}{\sqrt{\frac{15.6\text{ N}}{0.00863\text{ kg/m}}}} = 6$$

12. [4 pts] As you walk up the Ave on a rainy Seattle day, you notice a small oil spill. A thin layer of oil sits on top of a puddle of water. The index of refraction of oil is 1.48 and the index of refraction of water is 1.33. For the reflected waves 1 and 2, what colors in the visible spectrum (400 nm – 700 nm) will be absent from the oil film? Assume the light is initially incident normal to the surface and the oil film has a thickness of 700.0 nm.



A) 562 nm and 430 nm

B) 592 nm and 460 nm

C) 612 nm and 440 nm

D) 691 nm, 518 nm and 414 nm

E) 648 nm, 538 nm and 422 nm

The wave that reflects off of the top surface of the thin film will undergo a $\lambda/2$ phase shift (n increases) but the wave that reflects off of the bottom surface will not (n decreases). Thus, the condition for destructive interference is:

$$2nt = m\lambda$$

Rearranging for λ, we find:

$$\lambda = \frac{2nt}{m} = \frac{2(1.48)(700\text{ nm})}{m} = \frac{2072\text{ nm}}{m}$$

The first five wavelengths that would be absent from the reflection are as follows:

2072 nm, 1036 nm, ***691 nm, 518 nm, 414 nm****, 345 nm*

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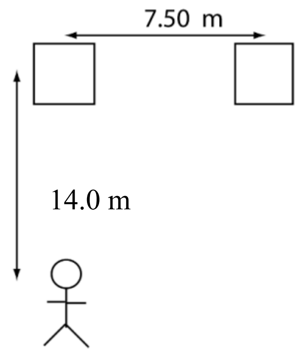
13. [4 pts] While attending a concert at the Neptune theater, you are sitting 14.0 m directly in front of the speaker on the left part of the stage. The speaker at the left part of the stage is positioned 7.50 m from the right speaker. v_{sound} = 343 m/s.



If the speakers are emitting a frequency of 410 Hz, will you hear a maximum sound, a minimum sound, or something in between?

A) Maximum sound

B) Minimum sound

C) Something in between

D) More information is needed.

We can find the distance between the student and the right speaker (l_r) using the Pythagorean theorem.

$$l_r = \sqrt{(14\text{ m})^2 + (7.5\text{ m})^2} = 15.88\text{ m}$$

We can now find the path length difference:

$$l_r - l_l = 15.88\text{ m} - 14\text{ m} = 1.88\text{ m}$$

We want to compare this path length difference to the wavelength of the sound wave.

$$\frac{\Delta l}{\lambda} = \frac{\Delta l}{\frac{v}{f}} = \frac{1.88\text{ m}}{\frac{343\text{ m/s}}{410\text{ Hz}}} = 2.25$$

Since $\Delta l = 2.25\lambda$, *the waves will neither completely destructively interfere or have maximum constructive interference, so the student will hear a sound between maximum and minimum*

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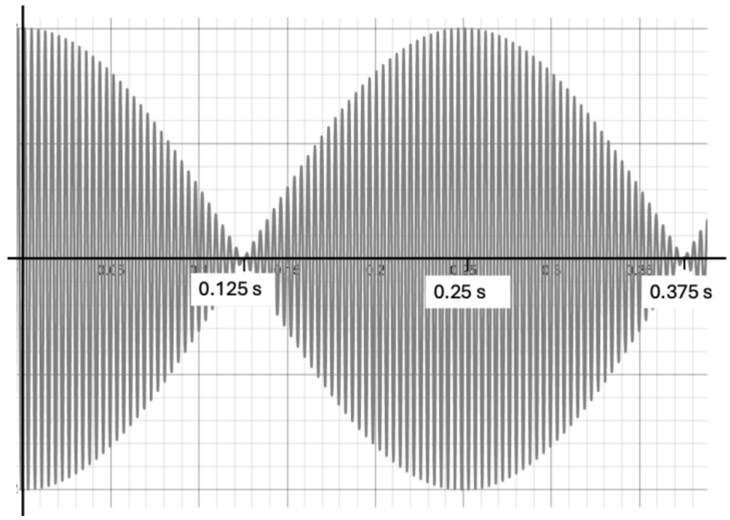
14. [3 pts] A student is tuning their guitar string by counting the beats when they pluck the string and simultaneously strike a tuning fork. The tuning fork has a known frequency of 256 Hz. The summation of the sounds produced by the guitar string and the tuning fork is shown at right. The student also notices that the pitch of the combined sounds is greater than 256 Hz. What is the frequency of the note produced by the guitar string?



A) 258 Hz

B) 260 Hz

C) 262 Hz

D) 264 Hz

E) 268 Hz

From the diagram, we can conclude that the time between beats is 0.25 s. This is the beat period which is the inverse of the beat frequency. The beat frequency is also the absolute value of the difference between the beats.

$$T_{beat} = \frac{1}{f_{beat}}$$

$$f_{beat} = |f_1 - f_2| = \frac{1}{T_{beat}} = \frac{1}{0.25\ \text{s}} = 4\ \text{Hz}$$

We now know the difference between the two frequencies. The tone that is heard by the student is the average of both frequencies ($f_{osc} = \frac{1}{2}(f_1 + f_2)$). Since the student hears a higher tone than 256 Hz, the string frequency must be higher than 256 Hz, and we know it must be 4 Hz lower. The string frequency is therefore 260 Hz.

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15. [4 pts] Your Physics instructor carries out a diffraction grating demonstration in class using a laser of wavelength λ. The screen is 6.50 m from the grating and the grating has 300 lines/mm. A student assistant measures the distance between the third bright fringe and the central maximum on the screen. They find the distance to be 3.70 m. Determine the wavelength of the laser.

A) 480 nm

B) 550 nm

C) 600 nm

D) 630 nm

E) 825 nm

The position of a bright fringe on a screen is given as:

$$y_m = Ltan\theta_m$$

We can arrange for θ_m *as follows:*

$$\theta_m = tan^{-1}\left(\frac{y_m}{L}\right)$$

The condition for bright fringes is given as:

$$dsin\theta_m = m\lambda$$

Rearranging for λ *and substituting for* θ_m*, we find:*

$$\lambda = \frac{d\ \sin\theta_m}{m} = \frac{d\ \sin\left(\tan^{-1}\left(\frac{y_m}{L}\right)\right)}{m} = \frac{\left(\frac{1\ \times\ 10^{-3}\ \text{m}}{300}\right)\sin\left(\tan^{-1}\left(\frac{3.70\ \text{m}}{6.50\ \text{m}}\right)\right)}{3} = 550\ \text{nm}$$

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Lab Multiple Choice Questions

16. [4 pts] Match the missing words in the sentences below to the list of words below the statement.

In Lab A1, you investigated standing waves on a string. The control variable was _(1)__. Possible options for the independent variable were ____(2)_____ and ____(3)______, and the dependent variable was ____(4)____.

a. Frequency of function generator
b. Number of antinodes
c. Linear mass density of the string
d. String tension

A) 1 matches to b, 2 and 3 match to either a or d, and 4 matches to c.

B) 1 matches to b, 2 and 3 match to either a or c, and 4 matches to d.

C) 1 matches to a, 2 and 3 match to either c or d, and 4 matches to b.

D) 1 matches to a, 2 and 3 match to either b or c, and 4 matches to d.

E) 1 matches to b, 2 and 3 match to either c or d, and 4 matches to a.

*In Lab A1, you examined standing waves on a string. You were asked to keep the number of antinodes constant throughout the experiment, thus making the number of antinodes the control variable (**1 matches to b**). You could then change the linear mass density of the string, the length of the string, or the tension in the string. These are the possible options for the independent variable (**2 and 3 match to either c or d**). The dependent variable is the frequency of the function generator (**4 matches to a**).*

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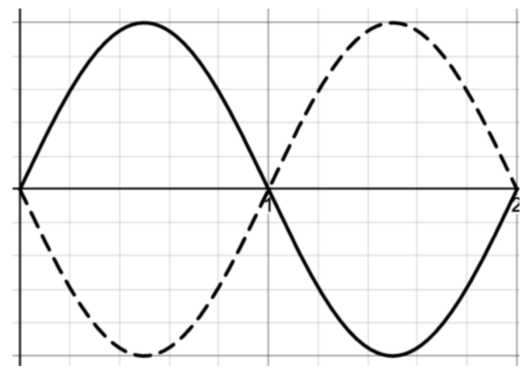
17. [4 pts] In Lab A1, two groups of students, group 1 and group 2 have adjusted the function generator to a frequency f_0 to form the standing wave at right. The string has a length L_0, and the mass of the hanger is m_0. Group 1 has decided to investigate how the standing wave frequency varies with length and group 2 is investigating how the standing wave frequency varies with mass.



Group 1 changes the length of the string to $2L_0$ and group 2 changes the mass of the hanger to $m_0/4$. How should the groups change the function generator to find the same standing wave pattern?

A) Group 1 should decrease the frequency to $f_0/2$, and group 2 should increase the frequency to $\sqrt{2}f_0$.

B) Group 1 should increase the frequency to $4f_0$, and group 2 should decrease the frequency to $f_0/\sqrt{2}$.

C) Group 1 should increase the frequency to $2f_0$, and group 2 should decrease the frequency to $f_0/\sqrt{2}$.

D) Group 1 should decrease the frequency to $f_0/2$, and group 2 should decrease the frequency to $f_0/2$.

E) Other

The frequency of a standing wave is given as follows:

$$f_m = m\frac{v}{\lambda} = m\frac{\sqrt{\frac{T}{\mu}}}{2l}$$

$$m = \frac{f_m 2l}{\sqrt{\frac{T}{\mu}}}$$

$$T = m_{hanger}g$$

The standing wave frequency is thus proportional to $\sqrt{T}$ and inversely proportional to the length of the string ($f \propto \sqrt{T}, f \propto \frac{1}{l}$). The tension in the string is proportional to the mass of the hanger.

To keep the same number of antinodes, group 1 should half the frequency to $f_0/2$, and group 2 should also decrease the frequency to $f_0/2$.

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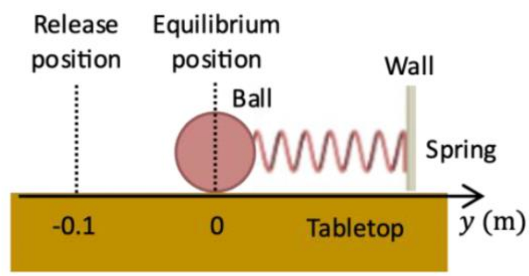
18. [4 pts] As part of Lab 2 homework and Lab A2, you examined the context at right. For this question, the ball has a mass 0.2500 kg and is resting on a frictionless tabletop. The ball is connected to one end of a spring with spring constant 5.000 N/m. The other end of the spring is attached to a wall that does not move. The position of the ball measured from equilibrium position is *y*. The ball is pulled 0.1000 m in the negative *y*-direction from the equilibrium position, and at time *t* = 0 s it is released from rest.



Using the same assumptions as those stated in the Phys 123 lab homework, what is the position of the ball at *t* = 0.01 s?

A) -0.09978 m

B) -0.09980 m

C) -0.09984 m

D) -0.09987 m

E) -0.09990 m

1. *We first calculate the net force on the ball when it is displaced 0.1 m from equilibrium.*

$$F_{net} = F_{by\ spring} = k\Delta l = (5\ \mathrm{N/m})(0.1\ \mathrm{m}) = 0.5\ \mathrm{N}$$

2. *We then use this force and Newton's second law to calculate the acceleration:*

$$a = \frac{F_{net}}{m} = \frac{0.5\ \mathrm{N}}{0.25\ \mathrm{kg}} = 2\ \mathrm{m/s^2}$$

3. *We then assume this acceleration is constant and find the speed at the end of the small time interval (0.01 s).*

$$v_f = v_i + at = (0\ \mathrm{m/s}) + (2\ \mathrm{m/s^2})(0.01\ \mathrm{s}) = 0.02\ \mathrm{m/s}$$

4. *We then consider the ball having this speed for the small-time interval and find its position.*

$$x_f = x_i + vt = -0.1\ \mathrm{m} + (0.02\ \mathrm{m/s})(0.01\ \mathrm{s}) = -0.09980\ \mathrm{m}$$

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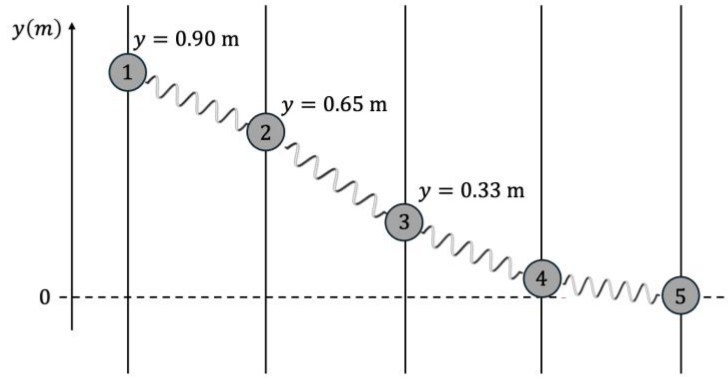
19. [3 pts] For the simulation of the five balls in Lab A2, balls 2, 3, and 4 are free to move and the net force on each of these balls is due to the springs to the left and right. Assume that the force from the left spring on ball 2 only depends on the difference of y positions of balls 1 and 2. Likewise, assume the force from the right spring on ball 2 only depends on the difference in the y positions of balls 2 and 3.



As noted in Q18, the balls have a mass of 0.250 kg, and the spring constant of each spring is 5.00 N/m. Ball 1 is displaced at $t = 0$ s, and the displacement of balls 1, 2, 3 at $t = 1.0$ s is shown above. What is the magnitude of the acceleration of ball 2 at this instant?

A) $1.4\ \mathrm{m/s^2}$

B) $0.78\ \mathrm{m/s^2}$

C) $5.1\ \mathrm{m/s^2}$

D) $2.3\ \mathrm{m/s^2}$

E) $11\ \mathrm{m/s^2}$

1. *We first calculate the net force on ball when it is displaced 0.1 m from equilibrium.*

$$\vec{F}_{net} = \vec{F}_{by\ left\ spring} + \vec{F}_{by\ right\ spring}$$

$$F_{net} = k(y_1 - y_2) - k(y_2 - y_3)$$

$$F_{net} = k[(y_1 - y_2) - (y_2 - y_3)] = (5\ \mathrm{N/m})[(0.9\ \mathrm{m} - 0.65\ \mathrm{m}) - (0.65\ \mathrm{m} - 0.33\ \mathrm{m})]$$

$$F_{net} = -0.35\ \mathrm{N}$$

2. *We then use this force and Newton's second law to calculate the acceleration:*

$$a = \frac{F_{net}}{m} = \frac{-0.35\ \mathrm{N}}{0.25\ \mathrm{kg}} = -1.4\ \mathrm{m/s^2}$$

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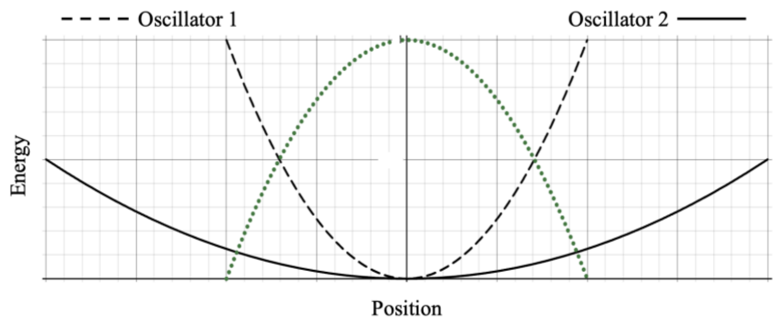
Lecture Free Response

The graphs at right show the potential energy of two separate SHM mass-spring oscillators as a function of their position. Oscillator 1 consists of block 1 of mass, m_1 and spring 1 with spring constant k_1. Oscillator 2 consists of block 2 of mass, m_2 and spring 2 with spring constant k_2.



20. [4 pts] On the graph at right, draw the kinetic energy curve for oscillator 1. Your curve should pass through at least 5 quantitatively correct data points.

The correct x-y coordinates (in terms of major gridline divisions) are (-2, 0), (-1.4, 1), (0, 2), (1.4, 1), (2, 0).

When the potential energy is maximum, the kinetic energy is minimum, and vice versa (this accounts for the coordinates (-2,0), (0,2) and (2,0).

You can choose any other two coordinates that satisfy the equation $E_{total} = K + U$.

21. [3 pts] Is the spring constant of oscillator 1 *greater than, less than,* or *equal to* the spring constant of oscillator 2? If it's not possible to tell, state so explicitly. Explain.

Greater than*. The spring potential energy of the spring is given as* $\frac{1}{2}kx^2$. *If we consider the spring potential energy at equal positions (or equal changes in length of the springs), we notice that the spring potential energy of oscillator 1 is greater than that of spring 2. It can therefore be concluded that the spring constant of spring 1 is greater than that of spring 2.*

We can also quantitatively compare the spring constants. For two distance units (major x-axis gridlines), the potential energy stored in spring 1 is 2 energy units (major y-axis gridlines).

$$\frac{1}{2}k_1(2)^2 = 2$$

$$k_1 = 1$$

For four distance units (major x-axis gridlines), the potential energy stored in spring 2 is 1 energy unit (major y-axis gridlines).

$$\frac{1}{2}k_2(4)^2 = 1$$

$$k_2 = \frac{1}{8} = \frac{k_1}{8}$$

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22. [4 pts] Suppose that $m_1 = 2m_2$. Is the maximum speed of block 1 *greater than, less than,* or *equal to* the maximum speed of block 2? If it's not possible to tell, state so explicitly. Explain.

Equal to: The maximum kinetic energy of block 1 is twice as large as the maximum kinetic energy of block 2, since $E_{total,1} = 2E_{total,2}$ and $E_{total} = K_{max}$. For block 1 to have twice the maximum kinetic energy as block 2 while also being double the mass of block 2, the two blocks must have the same maximum speed.

Alternatively.

$$E_{max,1} = \frac{1}{2}m_1 v_{1,max}^2 = 2E_{max,2} = 2\left(\frac{1}{2}m_2 v_{2,max}^2\right)$$

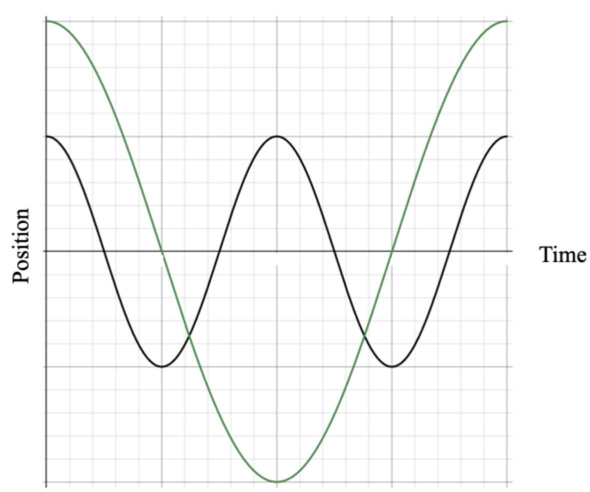
$$m_1 v_{1,max}^2 = 2m_2 v_{2,max}^2$$

$$m_1 = 2m_2$$

$$2m_2 v_{1,max}^2 = 2m_2 v_{2,max}^2 \qquad v_{1,max}^2 = v_{2,max}^2$$

$$v_{1,max} = v_{2,max}$$

23. [4 pts] The position of **block 1** as a function of time is shown at right. On the same graph, make a plot of the position of block 2 (assume block 2 is released at its maximum positive displacement at $t = 0$ s). Your graph should have 5 quantitatively correct data points. Briefly show your work.



We can write the period of oscillator 2 as follows:

$$T_2 = 2\pi\sqrt{\frac{m_2}{k_2}}$$

We also know from above, $m_1 = 2m_2$ *and* $k_1 = 8k_2$.

$$T_2 = 2\pi\sqrt{\frac{m_2}{k_2}} = 2\pi\sqrt{\frac{m_1}{2}\frac{8}{k_1}} = 2\pi\sqrt{4\frac{m_1}{k_1}} = 2\left(2\pi\sqrt{\frac{m_1}{k_1}}\right)$$

$$T_2 = 2T_1$$

And from the energy diagram above, $A_2 = 2A_1$.

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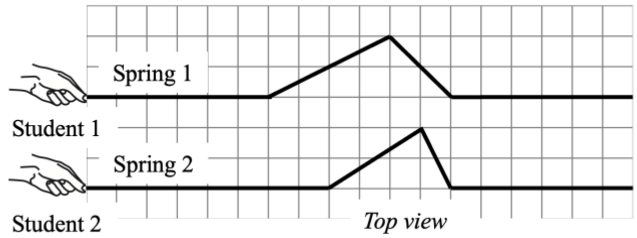
Tutorial Free Response Questions (All questions are independent of one another)

24. [4 pts] Two students generate two pulses on two tightly coiled springs by moving their hands up and down, as shown at right. The two students *finish* generating the pulses at exactly the same time.



Is the speed of the pulse on Spring 1 *greater than*, *less than* or *equal to* that of the pulse on Spring 2? Explain.

The two students finish generating the pulses at the same time. This means the trailing ends are created at the same time. The trailing edge of pulse 1 has moved a shorter distance (6 boxes) from the hand than the trailing edge of pulse 2 has moved in the same amount of time (8 boxes from the hand).

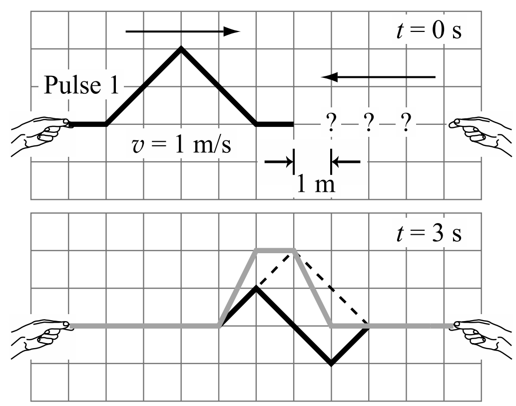
Since pulse 1 has traveled a shorter distance in the same amount of time, the speed of the pulse in spring 1 is less than that of the pulse in spring 2.

25. [3 pts] Pulses 1 and 2 are created on opposite ends of a spring. Pulse 1 travels at a speed of 1 m/s. (Each block is 1 m wide.) The top figure at right shows the shape and location of pulse 1 at time $t = 0$. Pulse 2 is not shown.

The shape of the entire spring at $t = 3$ s is shown in gray in the bottom figure at right.

In the bottom figure, clearly indicate the shape and location of pulse 2 at $t = 3$ s.

[No explanation required]



Sketch pulse 2 at time $t = 3$ s.

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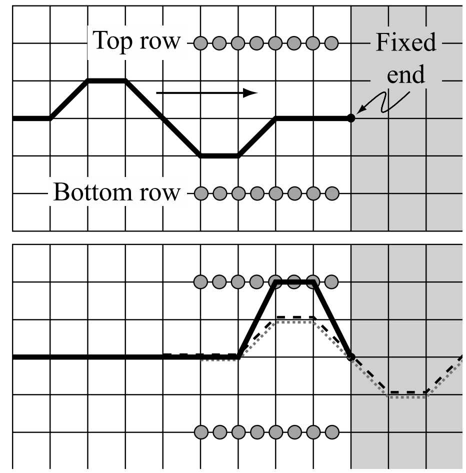
26. [4 pts] Two rows of cups are aligned on either side of a spring with a *fixed* end. The top figure at right shows the shape of the spring before the pulse reaches the fixed end.

    Will cups from the *top row, bottom row, both rows,* or *neither row* be knocked over? [*Note:* You do not need to indicate precisely which cups are knocked over, only from which rows, if any. Space has been provided to sketch the spring if you wish to do so.]

    Explain your reasoning.



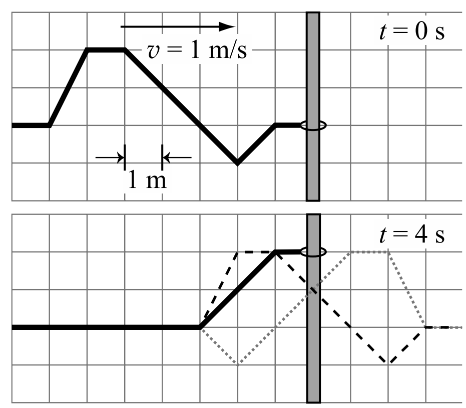
    Top row. A fixed-end reflection can be modeled as the superposition of the incident pulse and an imaginary pulse such that the boundary condition at the fixed end is satisfied. The spring will have the shape shown (solid black) in the bottom diagram when the incident pulse (dashed black) and imaginary pulse (dashed gray) are located at the indicated positions. As shown, cups in the top row will be knocked over. The incident and imaginary pulse will not cause a displacement of two units toward the bottom row on the left side of the fixed end. [They would, however, cause such a displacement on the right (imaginary) side of the fixed end.] ***Thus, only cups from the top row will be knocked over***

27. [4 pts] A pulse traveling at 1 m/s approaches the *free* end of a spring, as shown in the top figure at right.

    Sketch the shape of the spring at time t = 4 s in the space provided.

    Explain your reasoning.

    A free-end reflection can be modeled as the superposition of the incident pulse and an imaginary pulse. The incident pulse (dashed black) and imaginary pulse (dashed gray) are located as shown at t = 4 s. The shape of the spring is shown in black.



Sketch shape of spring at time t = 4 s