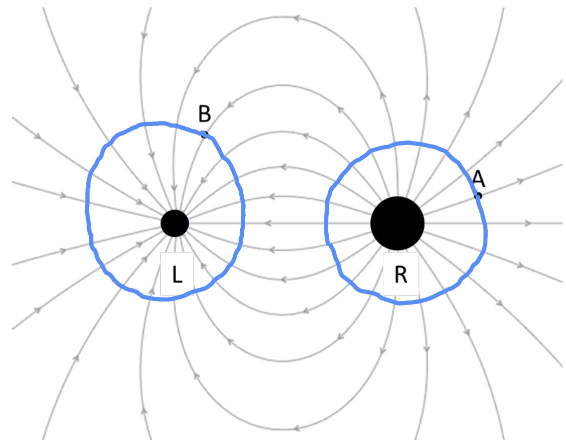
1. C
2. C
3. A
4. D
5. E
6. A
7. C
8. E
9. B
10. B
11. C
12. D

III. Lecture free response (25 points)

For all the lecture free response questions assume that the electric potential is zero at infinity.

Consider two solid metal spheres, as shown. The left sphere has radius R_0 and net charge $-Q_0$. The right sphere has radius $2R_0$ and net charge $+Q_0$. The electric field lines are shown. The potential at the surface of the left sphere is V_L and the potential at the surface of the right sphere is V_R.



13) (6 pts) Sketch **two** equipotential surfaces; one that passes through point A, and one that passes through point B.

14) (7 pts) What is the capacitance for this charge configuration? Write an expression in terms of variables given and constants.

$Q = C\Delta V_C$ $\quad \Delta V_C = V_R - V_L$ $\quad Q = Q_0$

$$\Rightarrow \boxed{C = \frac{Q_0}{V_R - V_L}}$$

15) (7 pts) A new particle with mass m_p and charge $+q_\mathrm{p}$ is added to the system. The new particle is initially at the surface of the left sphere and moving towards the right sphere with speed v. What is the minimum speed the particle must have if it is to reach the right sphere without turning around? Write an expression in terms of variables given and constants. Show your work.

$U_i + K_i = U_f + K_f$ Conservation of energy

Kinetic energy equation $K_i = \frac{1}{2} m_p v^2$ $\quad K_f = 0$ $\quad U = qV$ K_f=0 if the particle is to just make it.

$$\frac{1}{2} m_p v^2 = q_p (V_R - V_L) \Rightarrow \boxed{v_{min} = \sqrt{\frac{2(V_R - V_L) q_p}{m_p}}}$$

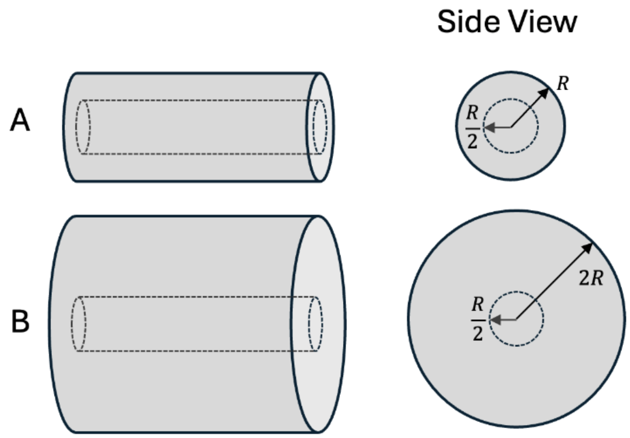
Now a conducting wire is connected between the two spheres.

16) (5 pts) When electrostatic equilibrium is reached, what is the potential at the surface of the left sphere? Write an expression in terms of variables given and constants. Explain your reasoning.

The initial total charge on the system is Q-Q=0. When the wire is connected charge can flow from one sphere to the other, but the total charge cannot change. Charge will flow until both spheres are at the same potential to reach the condition for electrostatic equilibrium. The potential on each sphere is proportional to the charge on the sphere, so charge -Q will travel from the left sphere to the right sphere. Then the charge on each sphere will be zero, so the total charge is still zero, and the potential on each sphere will be zero, so they will be equal to each other and proportional to the charge.

V=0

The diagram at right shows two infinitely long, solid non-conducting cylinders, cylinder A with radius R and cylinder B with radius $2R$. Each cylinder carries a total positive charge $+Q$ uniformly spread throughout their volumes. Each cylinder also has an imaginary Gaussian surface, a coaxial cylinder of radius $\frac{R}{2}$ and length L, centered inside.



1. [3 pts] Is the volume charge density for cylinder A *greater than*, *less than*, or *equal to* the volume charge density for cylinder B? Explain your reasoning.

   Greater than*. The volume charge density is the charge distributed throughout the containers volume divided by the volume of the container.*

$$\rho = \frac{Q}{V}$$

   Both cylinders have the same amount of charge throughout their volumes, but since cylinder A has a smaller volume that than of cylinder B, cylinder A has a larger volume charge density.

2. [4 pts] Is the net electric flux through the Gaussian surface within cylinder A *greater than*, *less than*, or *equal to* the net electric flux through the Gaussian surface within cylinder B? Explain your reasoning

   The net electric flux through a Gaussian surface is given as:

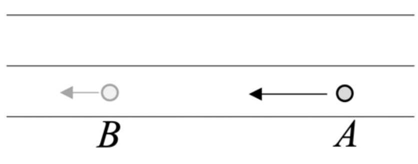
$$\Phi_E = \frac{Q_{in}}{\epsilon_0}$$

   The amount of charge enclosed in the Gaussian surface is equal to the product of the volume charge density and the volume of the Gaussian surface.

$$Q_{in} = \rho V$$

   Since the volume of each Gaussian cylinder is the same, and cylinder A has a greater volume charge density, the charge enclosed by cylinder A is greater than that enclosed by cylinder B. This means that the net electric flux through the Gaussian surface in cylinder A is greater than that through the Gaussian surface in cylinder B.

An electron is present in a unform horizontal electric field (direction of field is not shown). At point A, the electron has a speed v_A and is moving to the left. At point B, the electron has a speed v_B, where $v_B < v_A$. No external forces act on the electron.



3. [4 pts] What is the direction of the electric field? Explain.

As the electron moves to the right, it slows down. This means that it accelerates to the right. To accelerate to the right, the field must exert a force on the electron to the right. Electric force and electric field are related as follows:

$$\vec{F} = q\vec{E}$$

Since the charge of the electron is negative, the force direction is opposite to the electric field direction. As the force is to the right, the electric field must point ***to the left****.*

We can also consider the motion of an electron when it is released from rest. When released from rest, electrons speed up in the direction opposite to the electric field. Since this electron is slowing down, it must be moving in same direction of the electric field. As the electron velocity is to the left, the electric field must be to the left.

4. [4 pts] Is the value of $V_B - V_A$ positive, negative or zero? Explain.

The relationship between the electric field and the electric potential is given as:

$$\Delta V = -\int_i^f \vec{E} \cdot d\vec{s}$$

Since the electric field is to the left, and the electron's displacement is to the left, the change in potential ($V_B - V_A$) is ***negative*** *based on the equation above.*

We could also consider the closed electron-field system. As the electron slows down, the kinetic energy of the system decreases. Since the change in total energy of the system is zero (closed system), the potential energy must be increasing. The electric potential is related to the electric potential energy as follows:

$$\Delta V = \frac{\Delta U}{q}$$

Since the change in electric potential energy is positive, and the charge is negative, the change in electric potential is ***negative****.*