

1. B
2. B
3. B
4. C
5. B
6. B
7. E
8. D
9. A
10. A
11. E
12. C
13. A
14. B
15. E
16. D
17. E
18. A
19. C

**III. Lecture Free Response (25 points total)**

Questions 20 and 21 are related to the following scenario.

There is both a **uniform** magnetic field  $\vec{B} = -B\hat{z}$  and a **uniform** electric field  $\vec{E} = E\hat{x}$ . These fields fill all of space.

You observe that a particle with positive charge  $+Q$  moves in a straight line with a constant velocity vector  $\vec{v} = v_y\hat{y}$ .

20. [6 pts] State the forces acting on the particle and the **direction** of those forces. **Explain briefly.** (You may ignore gravity.)

$\vec{F}_E = q\vec{E}$ , so there is a force from the electric field on the positively charged particle in the direction of  $\vec{E}$  (right).

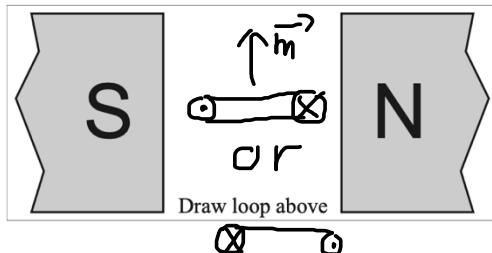
$\vec{F}_M = q\vec{v} \times \vec{B}$ . Using the right hand rule, there is a magnetic force on the particle that is to the left.

21. [8 pts] Use your answer to question 20 and your knowledge about the motion of the particle to determine  $v_y$  in terms of  $B$ ,  $E$ , and  $Q$ . Show your work and explain your reasoning.

The particle moves in a straight line with constant speed, so the acceleration is zero. By Newton's 2<sup>nd</sup> law, the net force on the particle should also be zero. So then  $|\vec{F}_E| = |\vec{F}_M|$  and we can write  $qE = qv_yB$ . Solving for  $v_y$  we get  $v_y = E/B$ .

22. [6 pts] A small current loop is placed between opposite poles of two large magnets as shown in the cross-sectional diagram at right.

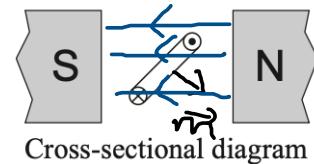
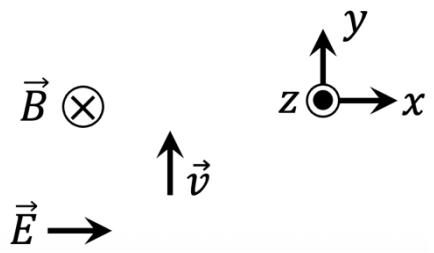
How must the loop be oriented so as to **maximize** the net torque? Sketch a cross-sectional diagram (like the one at right) to indicate your answer. Explain.



The torque on a magnetic dipole  $\vec{m}$  in an external magnetic field  $\vec{B}$  is given by  $\vec{\tau} = \vec{m} \times \vec{B}$ . The torque is largest when  $\vec{m}$  is perpendicular to  $\vec{B}$ .

$\vec{B}$  points from the north pole toward the south pole, so the magnetic moment for the loop should point either up or down. From the direction of  $\vec{m}$ , you can

determine the orientation of the current loop using the right hand rule.

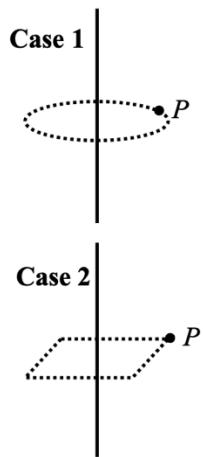


Cross-sectional diagram

23. [6 pts] You wish to calculate the magnetic field at point  $P$  near a very long current-carrying wire using Ampère's Law. In the two cases shown, two different closed loops, one square and one circular, are centered on the wire and pass through point  $P$ .

In which case(s), if any, could Ampère's Law be used to calculate the magnetic field at point  $P$ ? **Explain.** (You do not need to calculate the field.)

*Due to the symmetry of the current distribution, the magnetic field near the wire is directed in circles centered on the wire. In order to simplify the integral in Ampere's Law,  $\oint \vec{B} \cdot d\vec{s}$ , we need the angle between  $\vec{B}$  and the path direction to remain constant at all points on the path, and we need the field strength to be constant along the path. This is only true for **Case 1** (the circular loop).*



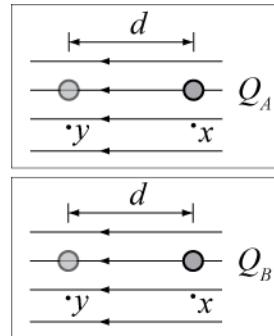
#### IV. Tutorial Questions (15 points total)

Two particles with the *same* positive charge,  $Q_A$  and  $Q_B$  are released from rest at point  $x$  in separate uniform electric fields that point in the negative  $x$ -direction. There is no interaction between the two charges. Both charges move through a distance  $d$  to the left. (Ignore any gravitational forces.)

24. [4 pts] Is the value of  $V_x - V_y$  *positive, negative, or zero*? If not enough information is given, state so explicitly. Explain.

**Positive.** The electric potential difference is given as  $\Delta V = -\int \vec{E} \cdot d\vec{s}$ .

This means that the electric potential decreases in the direction of the electric field. This means that  $V_y$  is less than  $V_x$ , and the difference  $V_x - V_y$  must be positive.



It is known that the mass of  $Q_A$  is half as large as the mass of  $Q_B$ . When both charges have moved from point  $x$  to point  $y$ , the speed of  $Q_A$  is measured to be twice the speed of  $Q_B$ .

25. [4 pts] Is the magnitude of the electric field strength in which particle A is present *greater than, less than, or equal to* magnitude of the electric field strength in which particle B is present? Explain.

**Greater than.** For uniform electric fields, the potential difference can be written as  $\Delta V = -\frac{\Delta K}{q} = -\vec{E} \cdot d\vec{s}$ . Since the angle between the displacement of the charges and the electric field is zero, we can write the absolute value of the dot product as  $E\Delta s$ . We can now write the electric field strength as:  $E = \Delta K/q\Delta s$ . Both  $Q_A$  and  $Q_B$  have the same positive charge and move through the same displacement. For a particle released from rest, the change in kinetic energy is given as  $\Delta K = \frac{1}{2}mv^2$ . With  $Q_A$  having the half the mass and twice the speed (and the speed is squared), the change in kinetic energy for  $Q_A$  is twice as large as that for  $Q_B$ . This means that  $Q_A$  is present in a stronger electric field than that of  $Q_B$ .

Two point charges are placed equidistant on either side of a neutral conducting sphere, as shown at right. The left charge is more positively charged than the right. Points  $L$  and  $R$  are locations that are equidistant from the center of the sphere.

Let  $\Delta V_{L \rightarrow R}$  be the electric potential difference from point  $L$  to point  $R$ .

26. [4 pts] Immediately after the point charges are placed (before charges in the conductor have had time to move), is  $\Delta V_{L \rightarrow R}$ , positive, negative, or zero? If there is not enough information, state so explicitly. Explain your reasoning.

*Negative.*

*Considering a positive test charge starting at point  $L$ , the rightward push from the  $+2Q$  is (on average) stronger than the leftward push from the  $+Q$ . Using work-energy theorem (e.g., external force must be leftwards and do negative work to move the charge at constant speed, the positive charge will gain kinetic energy if allowed to move freely, etc.) the change in potential energy will be negative. Since  $\Delta V = \frac{\Delta U}{q_{test}}$ , the change in potential must also be negative.*

27. [3 pts] The point charges are held in place as the charge distribution on the sphere stabilizes. After a long time, is  $\Delta V_{L \rightarrow R}$  positive, negative, or zero? If there is not enough information, state so explicitly. Explain your reasoning.

*Zero.*

*In equilibrium, the net electric field inside of a conductor is zero. Thus, no energy is needed to move a test charge from point  $L$  to point  $R$ , so the potential difference is zero.*

