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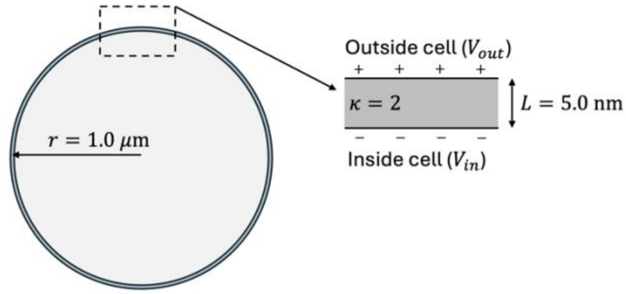
I. [60 pts] Multiple Choice (5 pts each): Mark your answer on BOTH the bubble sheet and this page.

For the next two questions, consider the electrophysiology of a ***spherical*** cell, of radius 1.0 µm. The cell is enclosed by a lipid bilayer membrane, thickness $L = 5.0$ nm with dielectric constant $\kappa = 2$. The membrane potential (*i.e.* the potential difference across the membrane) is $V_{in} - V_{out} = -220$ mV.



Hint: The cell acts like a capacitor with the cytoplasm (inside) and extra-cellular space (outside) acting like the conducting plates of a capacitor and the membrane as the dielectric. You can treat the membrane as a **parallel-plate capacitor**.
1 nm = 10^{-9} m.
1 µm = 10^{-6} m.

1. [5 pts] Determine the magnitude of the electric field in the membrane.

A. 1.1×10^{-9} V/m

B. 2.2×10^{7} V/m

C. 4.4×10^{7} V/m

D. 8.8×10^{7} V/m

E. 3.6×10^{10} V/m

We can write the magnitude of the electric field as follows:

$$|E| = \frac{|\Delta V|}{L} = \frac{220 \times 10^{-3}\ \text{V}}{5.0 \times 10^{-9}\ \text{m}} = 4.4 \times 10^{7}\ \text{V/m}$$

2. [5 pts] What is the net charge inside the cell, Q, in units of e, the **magnitude** of the charge of an electron?

A. $Q = -3.8 \times 10^{3}\ e$

B. $Q = -6.1 \times 10^{4}\ e$

C. $Q = -9.2 \times 10^{4}\ e$

D. $Q = 2.6 \times 10^{5}\ e$

E. $Q = -6.4 \times 10^{10}\ e$

We can use the relationship, $Q = CV$ *and* $C = \kappa C_0$*, to solve for the magnitude of the charge. We also note that the surface area of the capacitor is the surface area of the spherical cell (*$A = 4\pi r^2$*).*

$$Q = CV = \kappa C_0 V = \kappa \frac{\epsilon_0 A}{L} V = \kappa \frac{\epsilon_0 4\pi r^2}{L} V$$

$$Q = 2 \frac{(8.85 \times 10^{-12}\ \mathrm{C^2 \cdot N^{-1} \cdot m^{-2}}) 4\pi (1 \times 10^{-6}\ \mathrm{m})^2}{(5.0 \times 10^{-9}\ \mathrm{m})} (220 \times 10^{-3}\ \mathrm{V})$$

$$Q = 9.8 \times 10^{-15}\ \mathrm{C}$$

To express the charge in terms of e, we can divide Q by e (1.6×10^{-19} C*). And we should note that the charges/ions inside the cell are negative.*

$$Q = -6.1 \times 10^{-4}\ e$$

3. [5 pts] You want to design a capacitor-based energy storage device to power an electric car.

The breakdown electric field of air is 30.0 kilovolts per centimeter (kV/cm). Without a dielectric, calculate the volume of the smallest capacitor you could build that can store 1.0×10^3 J of electrostatic energy.

Hint: $u_E = U/volume$

A. 5.0 m^3

B. 2.5 m^3

C. 25 m^3

D. 62 m^3

E. 2.5×10^5 m^3

Note:

$$30\ \mathrm{kV/cm} = \frac{30 \times 10^3\ \mathrm{V}}{1\ \mathrm{cm}} = \frac{30 \times 10^3\ \mathrm{V}}{1\ \mathrm{cm} \times \frac{1\ \mathrm{m}}{100\ \mathrm{cm}}} = 3.0 \times 10^6\ \mathrm{V/m}$$

The volume charge density is given as:

$$u_E = \frac{U}{volume} = \frac{1}{2} \epsilon_0 E^2$$

We can therefore write the volume as:

$$volume = \frac{2U}{\epsilon_0 E^2} = \frac{2(1.0 \times 10^3\ \mathrm{J})}{(8.85 \times 10^{-12}\ \mathrm{C^2 \cdot N^{-1} \cdot m^{-2}})(3.0 \times 10^6\ \mathrm{V/m})^2}$$

$$volume = 25\ \mathrm{m^2}$$

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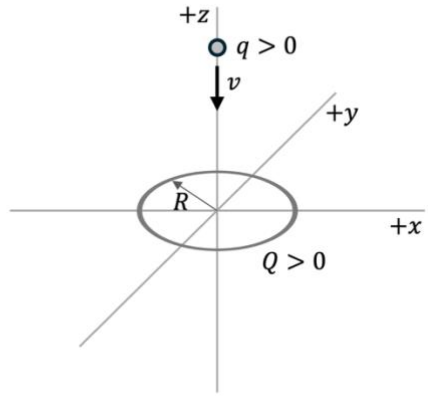
4. [5 pts] A uniformly charged ring, charge $Q > 0$ and radius R, is fixed in place in the *x*-*y* plane, centered at the origin. A particle, charge $q > 0$ and mass m, is fired along the +*z* axis from a great distance. What minimum speed is required for the particle to just pass through the loop assuming the particle travels along the +*z* axis?



A. $v \approx \sqrt{\frac{qQ}{2\pi\epsilon_0 Rm}}$

B. $v \approx \sqrt{\frac{qQ}{4\pi\epsilon_0 Rm}}$

C. $v \approx \frac{Q}{4\pi\epsilon_0 Rm}$

D. $v \approx \frac{qQ}{2\pi\epsilon_0 Rm}$

E. $v \approx \sqrt{\frac{Q}{4\pi\epsilon_0 Rm}}$

The potential of a charged ring with charge Q and radius R is given as:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{(z^2 + R^2)^{1/2}}$$

At the center of the ring, $z = 0$. This means the potential at the center of the ring is:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

We know that the particle is fired from a great distance where V = 0. This means it will move through a potential difference of $\frac{1}{4\pi\epsilon_0}\frac{Q}{R}$ when it reaches the center of the ring.

$$\Delta V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} = \frac{\Delta U}{q} = \frac{-\Delta K}{q}$$

$$\frac{1}{4\pi\epsilon_0}\frac{Q}{R} = \frac{-\frac{1}{2}m\left(v_f^2 - v_i^2\right)}{q}$$

We can assume that the final speed is almost zero as it passes through the loop ($v_f = 0$).

$$\frac{1}{4\pi\epsilon_0}\frac{Q}{R} = \frac{\frac{1}{2}m\left(v_i^2\right)}{q}$$

$$v_i = \sqrt{\frac{qQ}{2\pi\epsilon_0 Rm}}$$

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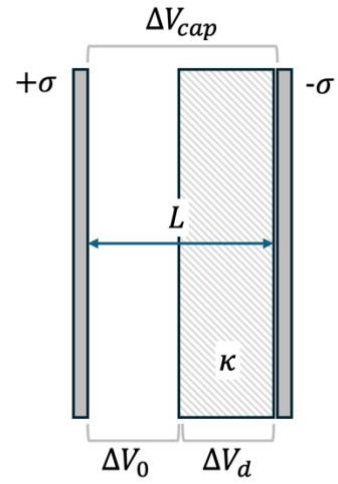
5. [5 pts] Consider a parallel-plate capacitor, with capacitance C, and a gap width L. A dielectric, with constant κ and width $\frac{1}{2}L$, is then inserted into the gap, half-filling it. The new capacitance is C'.

   Assume free charge densities $\pm\sigma$ on the plates of the half-filled capacitor. What is the magnitude of the potential difference across the plates, ΔV_{cap}?



   A. $\Delta V_{cap} = \frac{L\sigma\kappa}{2\epsilon_0(\kappa+1)}$

   B. $\Delta V_{cap} = \frac{L\sigma(\kappa+1)}{\epsilon_0\kappa}$

   C. $\Delta V_{cap} = \frac{L\sigma(\kappa+1)}{2\epsilon_0}$

   D. $\Delta V_{cap} = \frac{L\sigma(\kappa+1)}{\epsilon_0}$

   E. $\Delta V_{cap} = \frac{L\sigma(\kappa+1)}{2\epsilon_0\kappa}$

The voltage across the capacitor is equal to the sum of the voltage across the vacuum portion and the voltage across the dielectric.

$$\Delta V_{cap} = \Delta V_0 + \Delta V_d$$

The electric field of a vacuum capacitor is given as:

$$E_0 = \frac{\sigma}{\epsilon_0}$$

And the electric field of a dielectric is:

$$E_d = \frac{E_0}{\kappa}$$

We can now solve for the voltage across the capacitor.

$$\Delta V_{cap} = \Delta V_0 + \Delta V_d = E_0\frac{L}{2} + \frac{E_0}{\kappa}\frac{L}{2}$$

$$\Delta V_{cap} = E_0\frac{L}{2}\left(1 + \frac{1}{\kappa}\right) = E_0\frac{L}{2}\left(\frac{\kappa+1}{\kappa}\right)$$

$$\Delta V_{cap} = \frac{\sigma}{\epsilon_0}\frac{L}{2}\left(\frac{\kappa+1}{\kappa}\right) = \frac{L\sigma(\kappa+1)}{2\epsilon_0\kappa}$$

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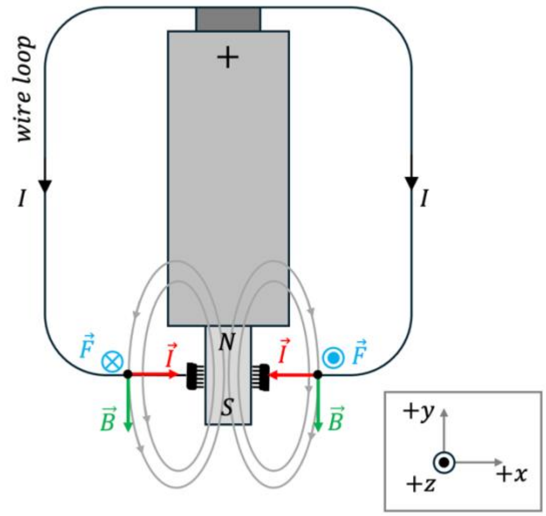
6. [5 pts] Consider the circuit shown in the schematic illustration to the right, constructed from a copper wire loop with conducting brushes, a conducting rare-earth magnet, and an AA battery. Some magnetic field lines of the magnet in the *x*-*y* plane are also shown. What does the circuit do?



   A. As viewed from above, the loop rotates clockwise around the *y*-axis.

   B. As viewed from above, the loop rotates counterclockwise around the *y*-axis.

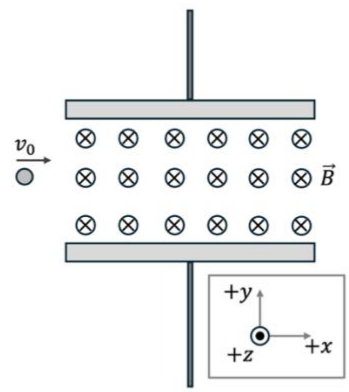
   C. It generates a force on the loop upwards.

   D. It generates a force on the loop downwards.

   E. No torques or forces are generated.

   The diagram above right considers the direction of the current and the magnetic field at two points on the left- and right- lower end of the wire loop. The RHR shows that the force on the right part of the loop is out of the page, and into the page on the left part of the loop. This will result in the loop rotating clockwise around the y-axis.

7. [5 pts] Consider the *velocity selector* shown in the schematic illustration to the right. What should the electric field be such that particles with magnitude of charge $|q|$ and velocity v_0 pass through the aperture undeflected? Assume $q \neq 0$.



   A. $\vec{E} = +v_0 B\hat{z}$

   B. $\vec{E} = -v_0 B\hat{z}$

   C. $\vec{E} = +v_0 B\hat{y}$

   D. $\vec{E} = -v_0 B\hat{y}$

   E. It depends on the sign of the charge

   If we begin with the assumption that the charge is positive, the magnetic force on the charge would point upward. To balance the magnetic force, the electric force would need to be downward. For a positive charge, the electric field must point downward or in the negative y-direction.

   If we consider the charge to be negative, the magnetic force would be downward. This means the electric force would need to be upward, which is still consistent with a downward pointing electric field.

$$\vec{F}_{elec} = \vec{F}_{mag}$$

$$q\vec{E} = qv\vec{B}$$

$$\vec{E} = v\vec{B}$$

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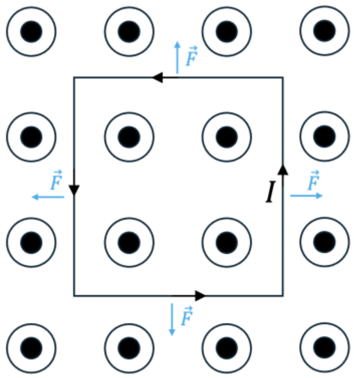
8. [5 pts] Consider the square wire loop that carries current I in the counter-clockwise direction, located in a uniform magnetic field B. What is the net force on the loop?



A. $\vec{F} = IAB\hat{z}$

B. $\vec{F} = -IAB\hat{z}$

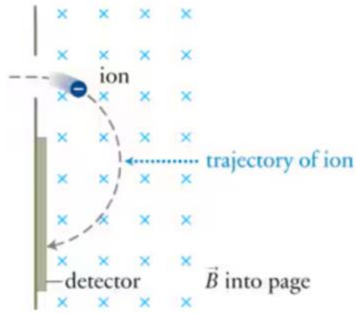
C. $\vec{F} = -(2-\pi)IRB(\hat{x}+\hat{y})$

D. $\vec{F} = (2-\pi)IRB(\hat{x}+\hat{y})$

E. $\vec{F} = 0$

The forces on each segment of wire are shown in the diagram above. The net force is zero.

9. [5 pts] Consider the *mass-spec apparatus* shown schematically in the illustration to the right. What is the time-of-flight T for a charge with velocity v to pass through the magnetic field and strike the detector?



A. $T = \frac{2\pi m}{qB}$

B. $T = \frac{\pi m}{qB}$

C. $T = \frac{qB}{mv}$

D. $T = \frac{mB}{qv}$

E. $T = \frac{\pi m B}{qv}$

The magnetic force on the charge is also the centripetal net force on the charge. We can therefore write the following equation:

$$F_{mag} = F_{cp} = ma_{cp}$$

$$qvB = m\frac{v^2}{r}$$

$$v = \frac{qBr}{m}$$

The charge follows a semi-circular arc of length πr. *The time-of-flight is therefore:*

$$\Delta t = T = \frac{distance}{speed} = \frac{\pi r}{\frac{qBr}{m}} = \frac{\pi m}{qB}$$

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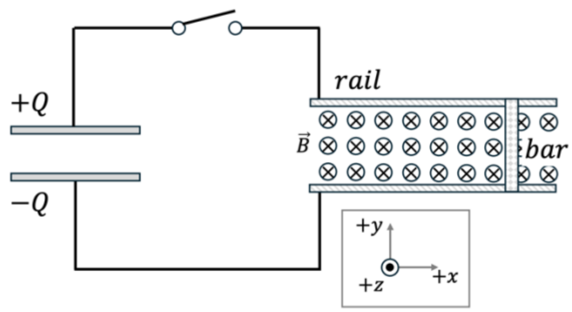
10. [5 pts] A circuit is constructed from a capacitor, a switch, two conducting rails and a conducting bar of width L. The bar sits across the rails and can move without friction. When the switch is closed, current flows from the capacitor, through the rails, and across the conducting bar. The goal is to have the bar move to the right. If the measured current is I and the magnitude of the force on the bar is F, what is the smallest-magnitude magnetic field that should be applied in the region containing the bar?



A. $\vec{B} = \frac{F}{IL}\hat{z}$

B. $\vec{B} = \frac{IL}{2F}\hat{z}$

C. $\vec{B} = -\frac{IL}{2F}\hat{z}$

D. $\vec{B} = -\frac{F}{IL}\hat{z}$

E. $\vec{B} = -\frac{IL}{F}\hat{z}$

Since the bar is made from a conducting material, it will carry a current in the negative y-direction when the switch is closed. The magnitude of the magnetic field that exerts a force on a current-carry wire is given by the following equation:

$$B = \frac{F}{IL\sin\theta}$$

The smallest magnetic field will occur when the field is perpendicular to the current, so we can write the smallest magnitude field as:

$$B = \frac{F}{IL}$$

For the force to be the right, the magnetic field must point into the page (or negative z-direction).

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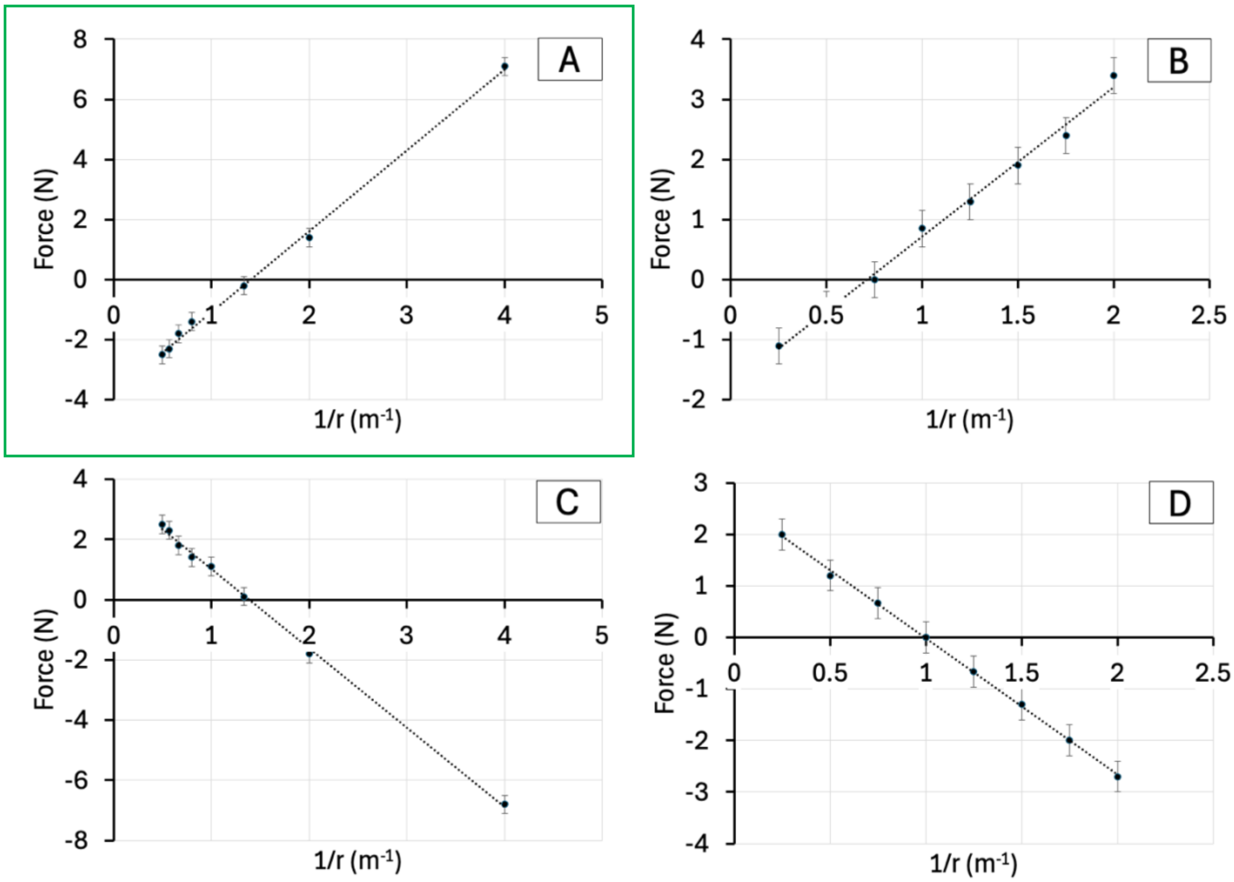
11. [5 pts] In Lab A3, students measured the force, F, on a minty particle, while varying the distance, r, from another minty particle. They begin with a separation distance of 0.25 m and increase in 0.25 m increments until 2.0 m, and notice that the particles are repelled when the separation is less than 0.75 m and attracted when the separation is greater than 0.75 m. They chose that a positive force on their force meter represents a repulsive force, and a negative force represents an attractive force. They notice that an F versus $1/r$ produces a linearized graph. Which graph (A to D) below is consistent with this description?



The students notice that the particles are repelled when the separation is less than 0.75 m, and they chose that repulsive forces are positive. The force measurements at 0.25 m, 0.50 m would correspond to values of 4 m^{-1} *and 2 m*$^{-1}$ *when considering the inverse of the distance (1/r). At these values, graph A has positive force measurements. Additionally, graph A highlights an equilibrium position at 1.33 m*$^{-1}$ *(or 1/0.75 m), and values less than 1.33 m*$^{-1}$ *are negative.*

Graph B is incorrect as it has its coordinates evenly spaced in terms of 1/r, where the students took measurements for equal spacing in r. Graph D has the incorrect equilibrium in addition to other errors. Graph C is consistent with the chosen system for positive and negative forces.

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12. [5 pts] In Lab A4, students take three force measurements between two exotic particles. The force measurement device is a digital scale that reads to one-hundredth of a newton. Their measurements are shown at right. If σ_r represents the random uncertainty and σ_i represents the instrumental uncertainty, what is the ratio σ_r/σ_i?

Trial	Force (N)
1	0.77
2	0.74
3	0.79

A. 2

B. 5

C. 8

D. 12

E. 15

The random uncertainty is determined by the standard deviation. The average of the data can be found as follows:

$$avg = \frac{0.77\text{ N} + 0.74\text{ N} + 0.79\text{ N}}{3} = 0.767\text{ N}$$

And the standard deviation as:

$$\sigma_r = \sqrt{\frac{(0.77\text{ N} - 0.767\text{ N})^2 + (0.74\text{ N} - 0.767\text{ N})^2 + (0.79\text{ N} - 0.767\text{ N})^2}{2}} = 0.025\text{ N}$$

The instrumental uncertainty for a digital scale is half the smallest increment.

$$\sigma_i = \frac{0.01\text{ N}}{2} = 0.005\text{ N}$$

The ratio is therefore:

$$\frac{\sigma_r}{\sigma_i} = \frac{0.025\text{ N}}{0.005\text{ N}} = 5$$

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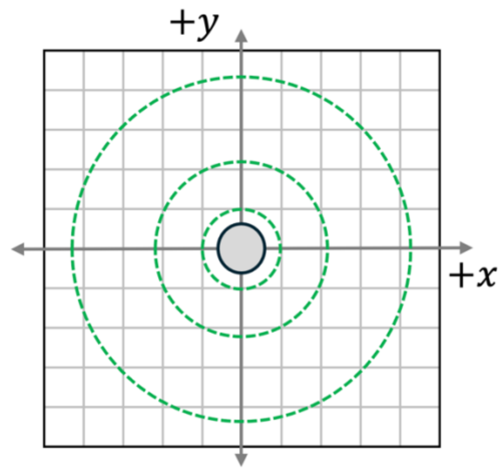
II. Lecture long-answer questions (20 points total)

Use this context for questions 13 to15 (Q16 is overleaf.) Consider an infinite-length wire with charge density λ, positioned along the z-axis as shown.



13. [5 pts] Draw **three** equipotential lines due to the wire in the x-y plane.

14. [5 pts] **Explain** whether the distance between the lines is *constant, increasing*, or *decreasing* with distance from the z-axis.

The electric field from an infinite-length wire with charge density λ decreases as a 1/r function, where r is the distance from the wire.

$$E = \frac{2k\lambda}{r}$$

Equipotential lines are more closely spaced where the electric field strength is large and more spaced out where the electric field is weak. Since the field decreases in strength with increasing distance from the wire, the spacing of the equipotential lines should increase with distance from the wire.

15. [5 pts] Using the charge density, set up an integral to calculate the electric potential at position x along the x-axis. Your answer should depend only on x, y, z, ϵ_0, and λ. You **do not** need to evaluate the integral. Show your work.

We could first consider an infinitesimal section of the wire with charge dq. The potential at a position x along the x-axis due to this segment could be written as:

$$dV = k\frac{dq}{r}$$

Where r represents the distance from the segment to position x.

Using the Pythagorean theorem, we can write the distance r in terms of z and x as follows:

$$r = \sqrt{x^2 + z^2}$$

We can also write the infinitesimal charge dq in terms of the charge density.

$$dq = \lambda dz$$

Integrating from negative infinity to positive infinity will yield the correct expression for the potential.

$$V = k\int_{-\infty}^{+\infty} \frac{\lambda}{\sqrt{x^2 + z^2}} dz$$

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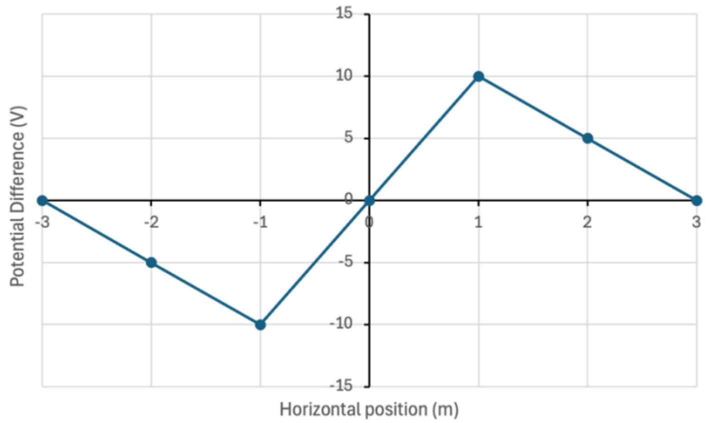
16. [5 pts] The figure at right shows a graph of *V* versus *x* in a region of space. The potential is independent of *y* and *z*. What is the electric field at *x* = 0 cm? Show your work.



The electric field can be written as follows:

$$\vec{E} = -\frac{\Delta V}{\Delta x}\hat{x}$$

We can use the values from the graph to determine the slope at x = 0 cm.

$$\vec{E} = -\frac{10\ \text{V}}{1\ \text{m}}\hat{x} = -(10\ \text{V/m})\hat{x}$$

This result shows that the electric field at x = 0 cm has a magnitude of 10 V/m and points in the negative x-direction.

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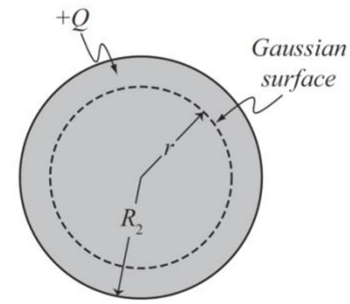
III. Tutorial and lab long answer questions (20 points total)

Two different non-conducting spheres of radius R_2 each have a total positive charge $+Q$ spread uniformly throughout their volumes. As shown at right, one sphere is solid. The other sphere is hollow, with a spherical cavity of radius $R_1 = \frac{1}{2}R_2$ at its center. Each sphere also has an imaginary Gaussian surface of radius $r = \frac{3}{4}R_2$ centered inside.



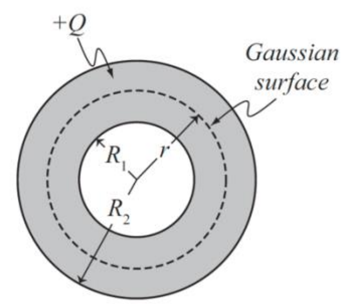
Cross-sectional view through center of solid sphere

17. [5 pts] Is the volume charge density for the hollow sphere *greater than*, *less than*, or *equal to* the volume charge density for the solid sphere? Explain your reasoning.

    Both spheres have uniform charge density. The hollow sphere has less volume to distribute charge over, so it has a greater charge density.

18. [5 pts] Is the net flux through the Gaussian surface within the hollow sphere *greater than*, *less than*, or *equal to* the net flux through the Gaussian surface within the solid sphere? Explain your reasoning.

    From Gauss's law, the net flux through a surface is equal to the charge enclosed. Both spheres do not enclose the charge from 3/4R to R, but the hollow sphere has a greater charge density, so there is more charge in that outer region. Thus there is less charge inside the Gaussian surface, so the net flux in the hollow sphere is less than the net flux in the solid sphere.



Cross-sectional view through center of hollow sphere

19. [5 pts] A particle moves from point P to point Q in a uniform electric field (electric field direction is purposely not shown). The charge moves through a positive change in potential and the work done by the electric field is negative. What is the sign of the charge? Explain your reasoning. Ignore gravity.

    For a uniform electric field, we can write the following equation:

$$\Delta V = -E_y \Delta y$$

    Since the charge moves in the negative direction and moves through a positive change in potential, we can conclude that the electric field points in the positive y-direction.

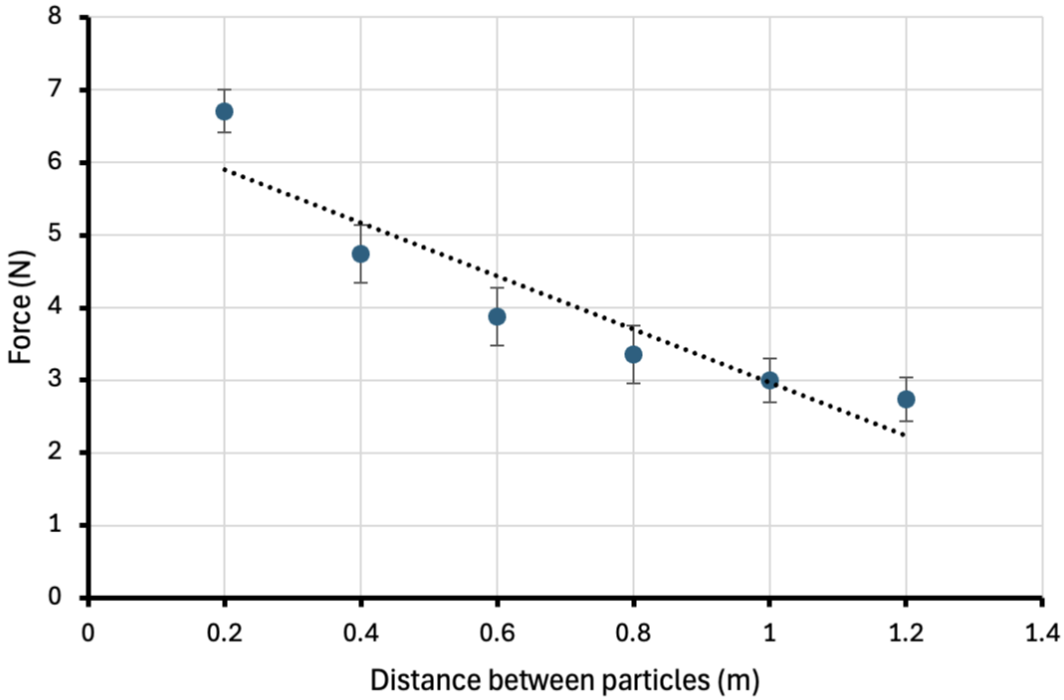
    The work done by the electric field can be written as:

$$W_{elec} = \vec{F}_{elec} \cdot \Delta\vec{y}$$

    Since the work done is negative, we can conclude that the electric force points in the positive y-direction. Since the electric force and electric field point in the positive y-direction, we can conclude that the charge is positive.

20. [5 pts] In a lab similar to Lab A4, a lab team explores the forces exerted between two exotic particles in a VR simulation. They make a plot of the force versus the distance between the two particles, as shown below.

Is the best-fit line a good-fit for the data? Explain.



For a best-fit line to be a good fit, the best line must pass through the error bars of at least 2/3 of the data points. Since the best-fit line above only passes through the error bars of 3 of the 6 data points, it is not a good fit.