1) A
2) C
3) D
4) B
5) C
6) C
7) D
8) B
9) D
10) E
11) E (We also accepted A and C, although in the lab workshop we stated that either I or III follow the rules we gave)
12) D

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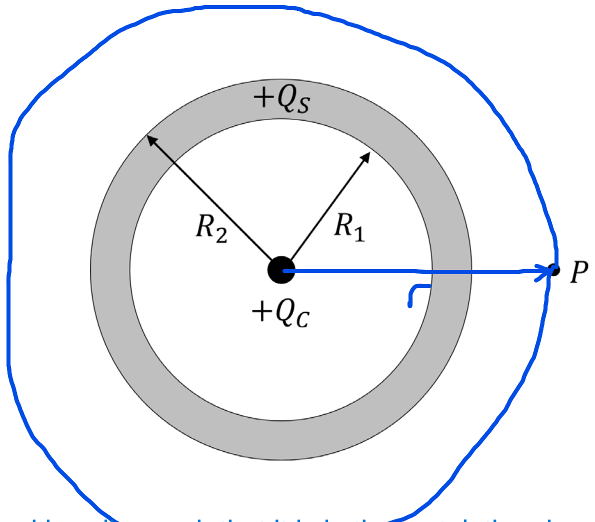
last *first*

III. Lecture free response (25 points)

Consider a spherical metal shell with outer radius R_2 and inner radius R_1 as shown. A particle with charge $+Q_C$ is located at the center of the sphere, and the metal shell has a net excess charge of $+Q_S$.



13) (6 pts) How much excess charge is on the inner surface of the metal shell? Answer in terms of the given variables. Be sure to include the sign and explicitly state if it is zero. Explain your reasoning.

$-Q_c$

If we consider a spherical Gaussian surface with radius such that it is in the metal, then because the metal is in electrostatic equilibrium the electric field in the metal is zero, so the flux through the surface is zero. Therefore total charge enclosed is zero. The total charge includes the charge in the center and the charge on the inner surface of the metal sphere.

14) (5 pts) How much excess charge is on the outer surface of the metal shell? Answer in terms of the given variables. Be sure to include the sign and explicitly state if it is zero. Explain your reasoning.

$Q_c + Q_s$

The initial charge on the metal is Q_S. Nothing touches this, so it does not change. The sum of the charge on the inner surface and outer surface must equal Q_S.

15) (5 pts) Draw a Gaussian surface that allows you to determine the electric field at point P and explain why you chose this surface.

To calculate the field at P the Gaussian surface must pass through P. The electric field at all points on the surface must be the same as that at P, so the Gaussian surface must have the same symmetry as the charge. The electric field is perpendicular to the surface and constant over the Gaussian surface drawn.

16) (3 pts) Write an expression for the area of your Gaussian surface. If your expression used any variables not already defined, be sure to indicate them on your drawing of the Gaussian surface.

$4\pi r^2$

17) (6 pts) Based on the Gaussian surface you drew, write an expression for the electric field at point P in terms of variables given, constants, and any variables you defined in question 16 above. Show your work and explain any assumptions.

Because of the symmetry of the Gaussian surface we drew, we can write the flux through the surface as EA, where E is the electric field at point P and A is the surface area given in 16). From Gauss's law the flux is given by

$$Q_{enc}/\varepsilon_0 = \frac{Q_c + Q_s}{\varepsilon_0}$$

Therefore

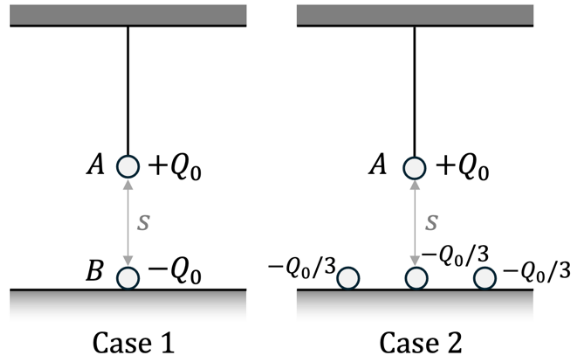
$$E\,4\pi r^2 = \frac{Q_c + Q_s}{\varepsilon_0} \quad \text{so} \quad \boxed{E = \frac{Q_c + Q_s}{4\pi\varepsilon_0 r^2}}$$

Tutorial Questions

1. [5 pts] Case 1: Consider two small, charged spheres, sphere A and sphere B. Sphere A has charge $+Q_0$ and sphere B has charge $-Q_0$. Sphere A is hung from a light inextensible string, and sphere B is placed on the ground directly below sphere A. The tension in the string is T_0.

   Case 2: Charge B is replaced with three small, charged spheres, each with charge $-Q_0/3$.

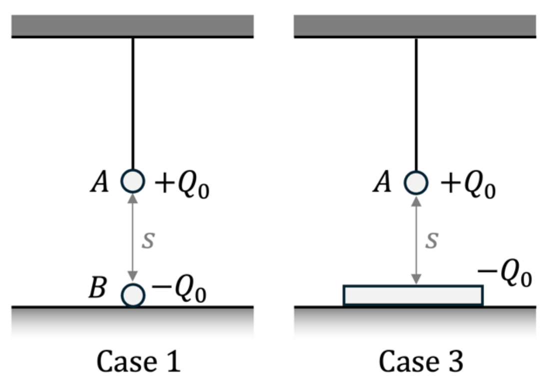
   Is the magnitude of the tension in the string in case 2 *greater than, less than,* or *equal to* T_0? Explain your reasoning.



   There are three forces exerted on sphere A in case 1, (1) a downward gravitational force (F_g) by the Earth, (2) a downward electric force (F_e) by sphere B, and (3) an upward tension force (T_0) by the string. Since sphere A is at rest, we can write: $T_0 = F_g + F_{e,1}$. In case 2, the small sphere directly below sphere A exerts a force on magnitude $F_e/3$ on sphere A since it has one-third of the charge of sphere B and it located the same distance s from sphere A. The two small spheres to the right and left of the center $-Q_0/3$ charge in case 2 exert a force on sphere A with magnitude less than $F_e/3$, since they are located a distance greater than s and $F_e \propto 1/r^2$. Additionally, in case 2, the x-components of the forces exerted by the left and right charges cancel due to the symmetry in their positions. As a result, the electric force on sphere A in case 2 is less than that in case 1. This means the tension in the string in case 2 is less than T_0.

2. [5 pts] In case 3, charge B from case 1 has been replaced with an insulating rod with charge $-Q_0$. The charge on the rod is uniformly distributed.

   Is the magnitude of the tension in the string in case 3 *greater than, less than,* or *equal to* T_0? Explain your reasoning.

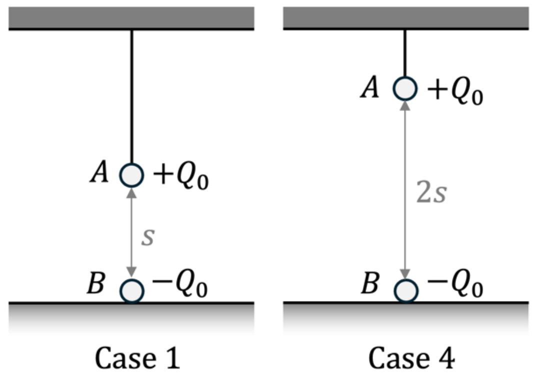


   The insulating rod in case 2 can be thought of as a continuous distribution of charge segments, each with a small amount of charge dQ. Each of the charge segments exerts an electric force on sphere A. For each segment on the right-half of the rod, we can pair it with a segment on the left-half of the rod such that the summation of forces exerted on sphere A by this pair would result in the cancellation of their x-components. Additionally, as we move farther from the middle of the rod, the force exerted by each charge segment weakens, as the distance to sphere A increases. As a result, the electric force exerted by the rod in case 2 is smaller in magnitude than the electric force exerted by sphere B in case 1. We can therefore conclude that the tension in the string in case 2 is less than T_0.

3. [5 pts] In case 1, it is known that the tension in the string (T_0), is twice the magnitude of the gravitational force exerted on sphere A

In case 4, the string attached to sphere A has been shortened such that the distance to sphere B is now $2s$.

In case 4, what is the magnitude of the tension in the string in terms of T_0? Explain your reasoning.



From question 18, we can state: $T_0 = F_g + F_{e,1}$. *In Q20, it states that the tension is equal to twice the magnitude of the gravitational force. We can therefore conclude that the electric force in case 1 is equal in magnitude to the gravitational force.*

$$T_0 = F_g + F_{e,1} = 2F_g$$

$$F_{e,1} = F_g$$

In case 4, the electric force is $F_{e,1}/4$ *since* $F_e \propto 1/r^2$. *Sphere A is at rest in both cases, so we can state:*

$$T_4 = F_g + F_{e,4} = F_g + \frac{F_{e,1}}{4}$$

Noting that $F_{e,1} = F_g$, *we can write:*

$$T_4 = F_g + \frac{F_g}{4} = \frac{5}{4}F_g$$

The last step is to write F_g *in terms of* T_0*:*

$$T_4 = \frac{5}{4}F_g = \frac{5}{4}\frac{T_0}{2} = \frac{5}{8}T_0$$