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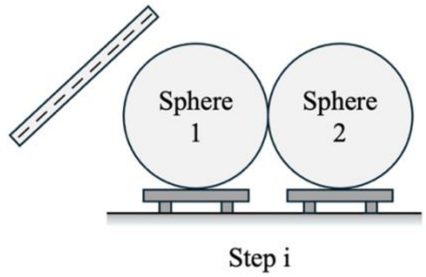
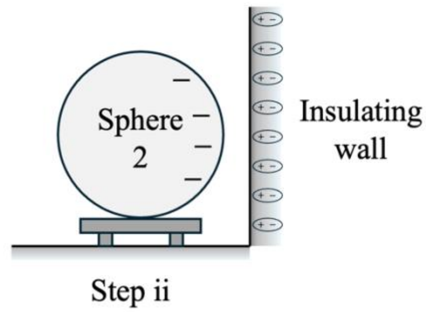
I. [60 pts] Multiple Choice (5 pts each): Mark your answer on BOTH the bubble sheet and this page.

1. [5 pts] Two identical and uncharged conducting spheres, 1 and 2, are initially in contact. The spheres are on insulating stands. Consider the following sequence:

   i. A negatively charged rod contacts sphere 1 and the rod is then removed.
   ii. Sphere 2 is then removed and brought into proximity but does not touch an insulating wall.





The electrostatic force on sphere 2 by the wall:

A. points away from the wall.

$\boxed{\text{B. points toward the wall.}}$

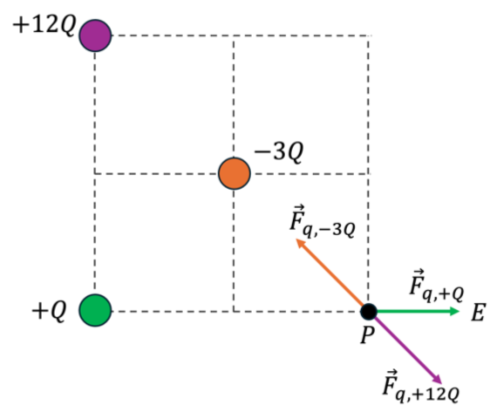
C. points upward.

D. is zero.

E. There is not enough information

From step 1, both spheres become negatively charged. A negatively charged sphere 2 is then brought close to the wall. At the atomic scale, the electron orbitals of the atoms in the wall becoming polarized, with the positive charge moving closer to the sphere 2. The closer distance of the positive part of the polarized atom to the sphere results in a net attractive force on the sphere that points toward the wall.

2. [5 pts] Three point charges ($+12Q, -3Q$ and $+Q$) are arranged as shown in the figure at right. A *positive* point charge is placed at point *P* in the lower right corner. Which arrow (A-E) best represents the direction of the net electric force on the *positive* point charge at point *P*?



A. Arrow A

B. Arrow B

C. Arrow C

D. Arrow D

$\boxed{\text{E. Arrow E}}$

The electrostatic force is given as: $F_{12} = k\frac{q_1 q_2}{r^2}$. *The +12Q charge is twice as far from point P than the -3Q charge, and the +12Q has four times the magnitude of charge as the -3Q charge. Due to the nature of the electrostatic force, the +12Q and the -3Q charges exert forces on the point charge at point P that are equal in magnitude and opposite in direction. These forces thus cancel, which leaves the repulsive force to the right exerted by the +Q charge.*

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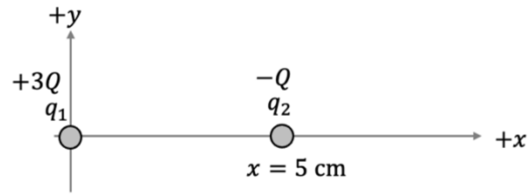
3. [5 pts] Two charges are placed along the x-axis, with charge $q_1 = 3Q$ at $x = 0$ and charge $q_2 = -Q$ at $x = 5$ cm. At what position should charge $q_3 = +25Q$ be placed such that the net force on q_1 is zero?



A. $x = 125$ cm

B. $x = 25$ cm

C. $x = -25$ cm

D. $x = 1.0$ cm

E. $x = -1.0$ cm

The -Q charge exerts an attractive force on the +3Q charge to the right. For the net force on the +3Q charge to be zero, it would need a force exerted to the left. Since q_3 is positive, q_3 must be placed to the right of q_1 to exert a repulsive force to the left. We can therefore rule out choices C and E.

Let's setup the force on q_1 by q_2 to be equal in magnitude to the force on q_1 by q_3, and the distance that q_3 is from the origin as r.

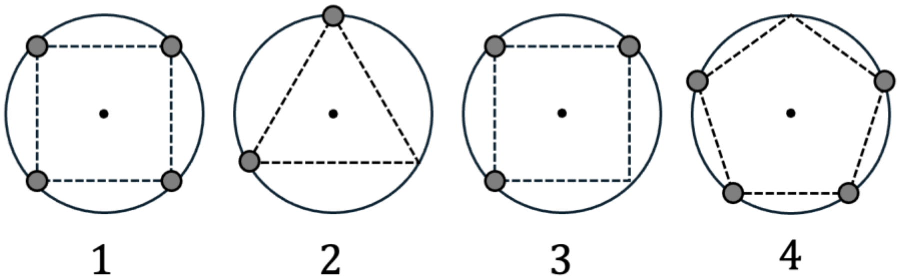
$$|F_{12}| = |F_{13}|$$

$$k\frac{q_1 Q}{(5\text{ cm})^2} = k\frac{q_1(25Q)}{(r)^2}$$

$$\frac{1}{25\text{ cm}^2} = \frac{25}{r^2}$$

$$r = \sqrt{25(25\text{ cm}^2)} = 25\text{ cm}$$

4. [5 pts] Consider the following configurations of point charges arranged around a circle at the corners of equilateral polygons. All charges have the same sign and magnitude, and some of the corners do not have a charge as shown. Rank the electric field magnitudes at the center of each circle (indicated by the small black dot), from smallest to largest. Treat each configuration as independent from the others.



A. $E_2 < E_3 < E_4 < E_1$

B. $E_1 < E_2 < E_3 < E_4$

C. $E_1 < E_4 < E_3 < E_2$

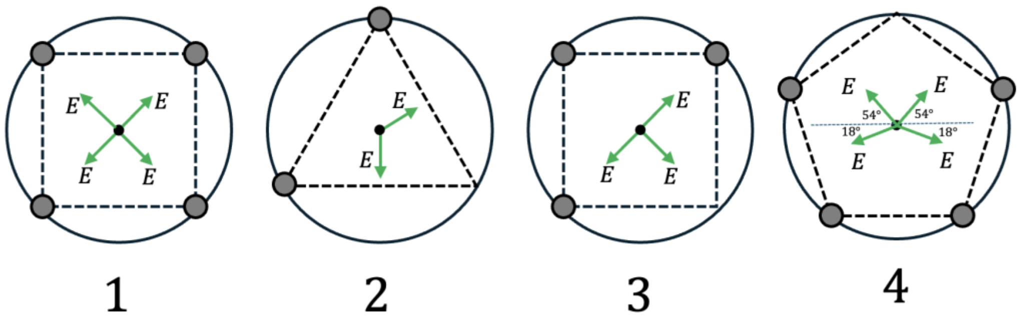
D. $E_2 = E_3 = E_4 < E_1$

E. $\boxed{E_1 < E_2 = E_3 = E_4}$

Due to symmetry, the electric field at the center in case 1 is zero. This means that we can rule out choices A and D.

If we add back in the missing charge in cases 2, 3, and 4, the electric field at the center would also be zero. This means that with the missing charge, the magnitude of the electric field at the center of cases 2, 3 and 4 are all equivalent to each other and equal to the electric field created by a single charge that is located on the circle. This might be most obvious in case 3, where the electric field due to the top right and bottom left charges would cancel, leaving just the field from the top left charge.

To consider a quantitative solution for case 2 and case 4, lets denote the electric field at the center of the circle due to a single charge located on the circle as E.



$$E_2 = \sqrt{{E_{2,x}}^2 + {E_{2,y}}^2}$$

$$E_2 = \sqrt{[E\cos 30°]^2 + [E\sin 30° - E]^2} = \sqrt{E^2(\cos 30°)^2 + E^2(\sin 30° - 1)^2}$$

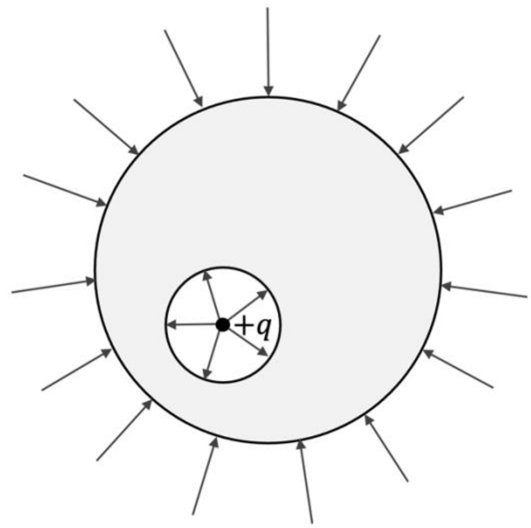
$$E_2 = E$$

$$E_4 = \sqrt{{E_{4,x}}^2 + {E_{4,y}}^2}$$

$$E_4 = \sqrt{0^2 + [2E\sin 54° - 2E\sin 18°]^2} = \sqrt{4E^2(\sin 54° - \sin 18°)^2}$$

$$E_4 = E$$

5. [5 pts] A conducting sphere has unknown charge Q. A known charge $+q$ is placed in a spherical void as shown above. What is the unknown charge Q?



A. $+q$

B. $-q$

C. $-3q$

D. $+4q$

E. $-4q$

$$Q_{unknown} = \frac{-20\ lines}{+5\ lines} q = -4q$$

It is important to count the lines both outside and inside and note the direction.

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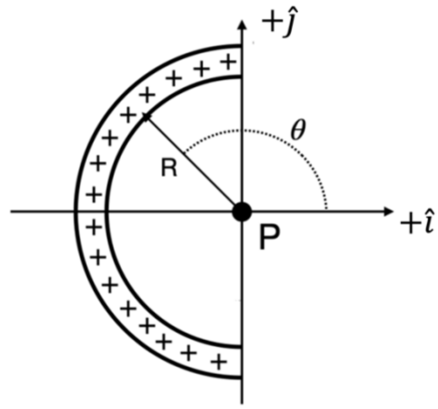
6. [5 pts] A thin semi-circular conductor carries a charge of $+Q$. The charge is uniformly distributed, and you can neglect the thickness of the conductor. Which of the following expressions of the electric field at point P is correct?



A. $\boxed{k\frac{Q}{\pi R^2}\int_{-\pi/2}^{\pi/2}\cos(\theta)\,d\theta\,\hat{\imath}}$

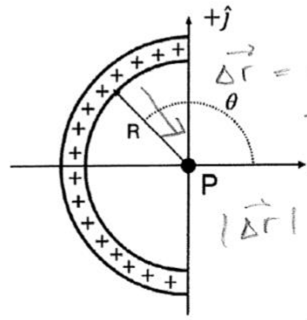
B. $k\frac{Q}{\pi R^2}\int_{-\pi/2}^{\pi/2}\sin(\theta)\,d\theta\,\hat{\jmath}$

C. $k\frac{Q}{\pi R}\int_{-\pi/2}^{\pi/2}\cos(\theta)\,d\theta\,\hat{\imath}$

D. $k\frac{Q}{\pi R^2}\int_{-\pi/2}^{\pi/2}\sin(\theta)\,d\theta\,\hat{\imath} + k\frac{Q}{\pi R^2}\int_{-\pi/2}^{\pi/2}\cos(\theta)\,d\theta\,\hat{\jmath}$

E. None of the above

Given the symmetry in the problem, we know that the electric field at point P must point in the $+\hat{\imath}$ direction. This means we can rule out choices B and D.



$\vec{\Delta r} = -x'\hat{x} - y'\hat{y}$

$|\vec{\Delta r}| = R$

$dq = R\,d\theta$

$\lambda = \frac{Q}{\pi R}$

$\frac{\hat{\Delta r}}{r^2}$

Sym.

0

$$\vec{dE} = k\,dq \cdot \frac{-x'\hat{x} - y'\hat{y}}{R^3}$$

$$= k\int_{\pi/2}^{3\pi/2} d\theta \; \frac{R\,Q}{\pi R} \; \frac{-R\cos\theta\,\hat{x} - R\sin\theta\,\hat{y}}{R^3}$$

$$= -\frac{kQ}{\pi R^2}\int_{\pi/2}^{3\pi/2} d\theta\,\cos\theta\,\hat{x}$$

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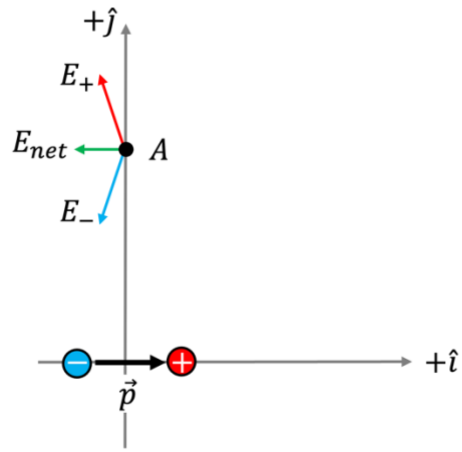
7. [5 pts] A dipole is centered on the origin as shown, with a dipole moment $\vec{p}$ that points along the x-axis. What is the electric field of the dipole at point A on the y-axis? Point A is far away from the dipole.



A. $\boxed{-k\frac{p}{|y|^3}\hat{\imath}}$

B. $+k\frac{p}{|y|^3}\hat{\imath}$

C. $-k\frac{2p}{|y|^3}\hat{\imath}$

D. $+k\frac{p}{|y|^3}\hat{\jmath}$

E. None of the above

The dipole moment vector points from the negative charge to the positive charge. Considering the diagram above, we can show that the electric field must point in the negative x direction.

The magnitude of the electric field far from a dipole along the bisector axis is: kp/r^3.

8. [5 pts] The electric force due to a uniform external electric field causes a torque of magnitude 10.0×10^{-9} N · m on an electric dipole oriented at 30.0 degrees from the direction of the external field. The dipole moment of the dipole is 7.00×10^{-12} C · m. What is the magnitude of the external electric field?

A. 200 NC^{-1}

B. 570 NC^{-1}

C. 1650 NC^{-1}

D. $\boxed{\text{2860 NC}^{-1}}$

E. None of the above

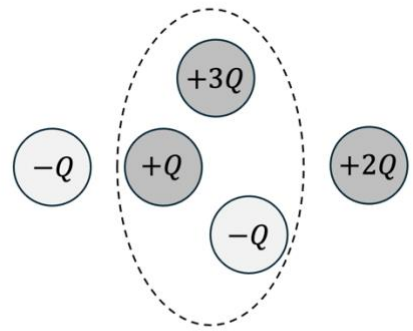
The torque exerted on a dipole in a uniform electric field is given as:

$$\tau = pE\sin\theta$$

We can solve for the electric field as follows:

$$E = \frac{\tau}{p\sin\theta} = \frac{10.0 \times 10^{-9}\ \text{N} \cdot \text{m}}{(7.00 \times 10^{-12}\ \text{C} \cdot \text{m})\sin 30°} = 2860\ \text{N/C}$$

9. [5 pts] In the electric field line diagram for the charge arrangement shown in the figure, eight field lines emanate from the object of charge $+1Q$. What is the field line flux through the closed surface indicated by the dashed line?



   A. -24

   B. +8

   C. +24

   D. +32

   E. Not enough information

   We can solve this problem as follows:

$$(3+1-1)Q\,\frac{8\,field\ lines}{+1Q} = +24\,field\ lines$$

10. [5 pts] A positively charged hollow sphere of radius 1.00 m has a uniform surface charge density of 10.0 nC/m^2. Determine the magnitude of the electric field at a distance of 1.20 m from the center of the sphere.

   A. 78 N/C

   B. 145 N/C

   C. 785 N/C

   D. 880 N/C

   E. None of the above

   We can solve this problem as follows:

$$E_{sphere} = k\frac{q}{r^2} = k\frac{\sigma A}{r^2} = (9\times 10^9\ \mathrm{N\cdot m^2/C^2})\frac{(10\times 10^{-9}\ \mathrm{C/m^2})4\pi(1\ \mathrm{m})^2}{(1.2\ \mathrm{m})^2} = 785\ \mathrm{N/C}$$

   Or we can use Gauss' law:

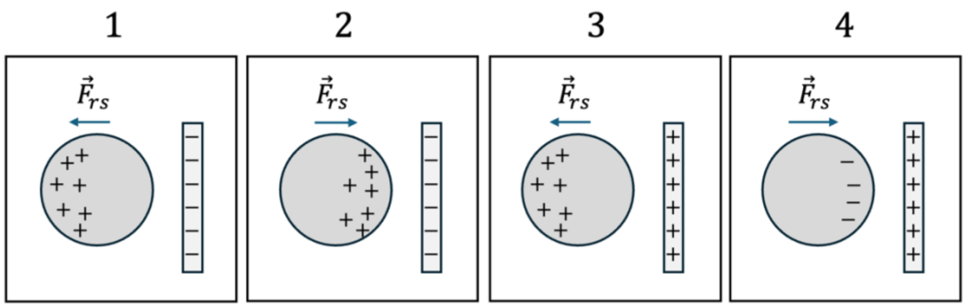
Use Gauss Law.

$$4\pi r^2 E\Big|_{r=1.2\,m} = \frac{4\pi R^2 \sigma}{\epsilon_0}\Big|_{R=1\,m}$$

$$E = \frac{\sigma}{\epsilon_0}\left(\frac{1}{1.2}\right)^2$$

11. [5 pts] An initially uncharged metal sphere is placed on a fulcrum, as shown. The metal sphere is charged by scraping an insulating rod along the surface of the sphere (the rod had previously been rubbed with a cloth). The rod is charged again and brought close to the sphere but does not touch the sphere. The type of material the rod and cloth are made of is not known.

Consider the four diagrams below which show possible excess charge distributions on the sphere and rod, in addition to the direction of the force by the rod on the metal sphere, $\vec{F}_{rs}$. The diagrams are top view diagrams. Which of these diagrams illustrates a physically possible scenario?



A. Diagram 1 only

B. Diagram 2 only

C. Diagram 3 only

D. Diagrams 1 and 3

E. Diagrams 2 and 4

The type of charge on the rod must match the type of charge on the sphere. This means that diagram 3 is the only correct choice from those shown above. The like charges on both the rod and sphere can result in a repulsive force.

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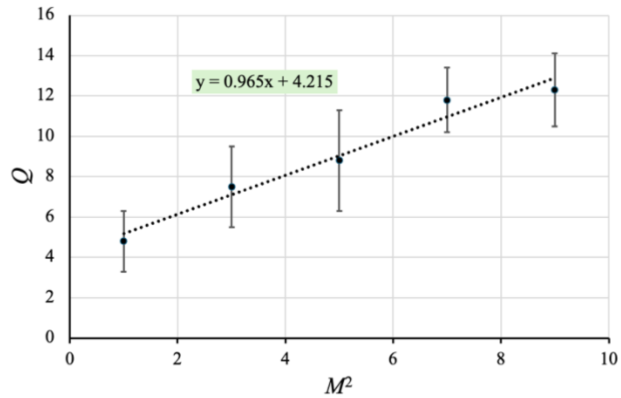
12. [5 pts] A student has formed the graph of quantity Q versus the square of quantity M (M^2). They have also drawn the best-fit line on the graph. Which of the following is true?



A. Quantity Q is proportional to quantity M.

B. Quantity Q is proportional to quantity M^2.

C. Quantity Q is linearly related to quantity M.

D. Quantity Q is linearly related to quantity M^2.

A proportional relationship between two variables, x and y, is given by the equation: $y = mx$. *A plot of a proportional relationship is a straight line that* ***goes through the origin***.

A linear relationship between two variables, x and y, is given by the equation: $y = mx + c$. *A plot of a proportional relationship is a straight line that has a non-zero intercept.*

In the plot above, the trendline is straight and has a non-zero intercept. This means that the quantity Q has a linear relationship with M^2. *The equation of the trendline is also showed, which we can write as:*

$$Q = 0.965M^2 + 4.215$$

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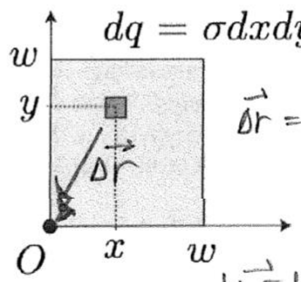
II. Lecture long-answer questions (20 points total)

13. [4 pts] Consider a uniformly-charged square, charge density σ and side length w, positioned as shown. Set up the integral to compute the E field vector at the origin O, but **do not compute it!**

    Provide the **numerators** and **denominators** for the integral below. Your answer should depend only on x and y. Show your work.



$\vec{\Delta r} = -x\hat{x} - y\hat{y}$

$|\vec{\Delta r}| = \sqrt{x^2+y^2}$

$$d\vec{E} = k\, dq \frac{\hat{\Delta r}}{\Delta r^2} = k\, dq \frac{\vec{\Delta r}}{\Delta r^3}$$

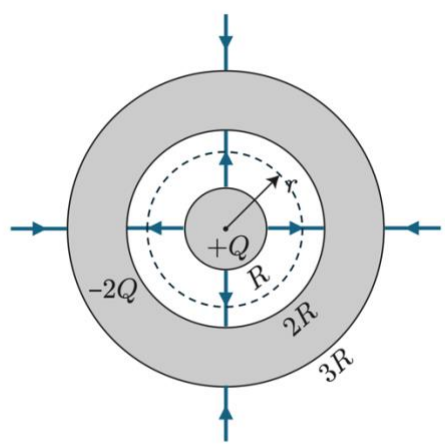
$$= k\sigma \frac{dx dy\, \vec{\Delta r}}{\Delta r^3}$$

$$\frac{\vec{\Delta r}}{|\Delta r|^3}$$

$$\vec{E}_O = k\sigma \int_0^w dx \int_0^w dy \frac{-x}{(x^2+y^2)^{3/2}}\hat{x} + \frac{-y}{(x^2+y^2)^{3/2}}\hat{y}$$

inside:

For the next **4** problems, consider a spherical conductor, charge $+Q$, in the center of a spherical shell, charge $-2Q$. The geometrical parameters are defined in the figure. Assume $Q > 0$.



14. [4 pts] Draw field lines (SOLID lines) representing the electric field.

15. [4 pts] Draw a Gaussian surface (DASHED lines) that can be used to compute the E field at radius r for $R < r < 2R$.

16. [4 pts] What is the charge density, σ, at $r = 3R$? Show your work for full credit.

$$Q_{in} + Q_{out} = -2Q$$

$Q_{in} = -Q$, such that $\vec{E} = 0$ inside the conductor.

$$Q_{out} = -Q$$

$$\sigma = \frac{Q_{out}}{4\pi r^2} = \frac{-Q}{4\pi r^2}$$

At $r = 3R$

$$\sigma = \frac{-Q}{4\pi(3R)^2} = \frac{-Q}{36\pi R^2}$$

17. [4 pts] Calculate the electric field at $r = 4R$. Show your work for full credit.

$$Q_{enc} = -2Q + Q = -Q$$

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = E(4\pi r^2)$$

$$At\ r = 4R, \qquad \Phi_E = E(4\pi(4R)^2) = E(4\pi 16R^2) = E(64\pi R^2)$$

$$\Phi_E = \frac{Q_{enc}}{\epsilon_0} = \frac{-Q}{\epsilon_0} \qquad and\ r = 4R$$

$$E(64\pi R^2) = \frac{-Q}{\epsilon_0}$$

$$E = \frac{-Q}{64\pi R^2 \epsilon_0}$$

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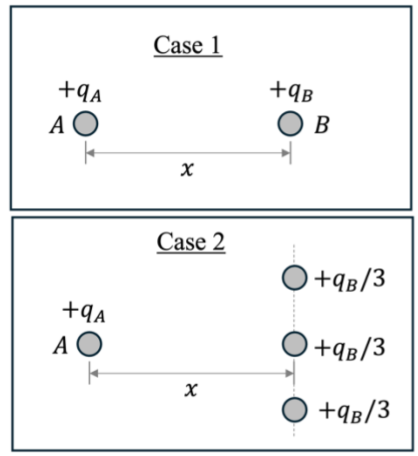
III. Tutorial and lab long answer questions (20 points total)

Use this context for questions 18 and 19.

Two experiments involve charged particles: In case 1, two positively charged particles, A and B are held a distance *x* apart as shown. The magnitude of the charge on particle A is *greater than* the magnitude of the charge on particle B ($|q_A| > |q_B|$).



In case 2, particle B is replaced by three identical particles of charge $q_B/3$ that lie along a line as shown.

18. [5 pts] In case 1, is the magnitude of the electric force on particle A *greater than, less than,* or *equal to* the magnitude of the electric force on particle B? Explain.

   The force that q_A exerts on q_B is a third-law force pair with the force that q_B exerts on q_A. This means that these forces are equal in magnitude.

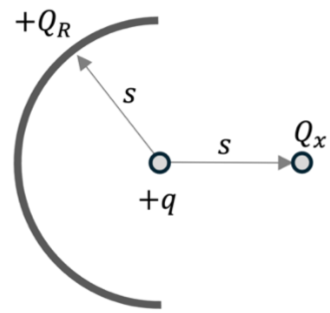
   We can also consider Coulomb's law and note that both charges appear in the numerator.

$$\left|\vec{F}_{q_A,q_B}\right| = \left|\vec{F}_{q_B,q_A}\right| = k\frac{q_A q_B}{x^2}$$

19. [5 pts] Is the magnitude of the net electric force on particle A in case 2 *greater than, less than,* or *equal to* the magnitude of the electric force on particle A in case 1? Explain.

   Less than. Each of the $+q_B/3$ charges in case 2 exert a force on the $+q_A$ charge that is either equal to or less than one-third of the force that the $+q_B$ charge in case 1 exerts on the $+q_A$ charge (due to Coulombs law). The y-components of the forces exerted by the upper and lower $+q_B/3$ charges in case 2 will cancel due to symmetry. The summation of the x-components of the upper and lower $+q_B/3$ charges with that of the middle $+q_B/3$ will therefore result in a smaller force on the $+q_A$ charge in comparison to the single force exerted by the $+q_B$ charge in case 1.

A thin semicircular rod has a total charge $+Q_R$ uniformly distributed along it. A positive point charge $+q$ and a charge (Q_x) of unknown sign and unknown magnitude are placed as shown. The $+q$ charge is equidistant from Q_x and from all points on the rod. It is known that the net force on the positive point charge $+q$ is zero.



20. [5 pts] What is the *sign* of the charge Q_x and is the magnitude of charge ($|Q_x|$) *greater than, less than,* or *equal to* the magnitude of the charge on the rod ($|Q_R|$)? Explain your reasoning.

   The rod will exert a force on the +q charge to the right, since they have the same sign of charge and the symmetry of the arrangement. For the net force on the +q charge to be zero, it must experience a force to the left by the Q_x charge. This would therefore be a repulsive force, meaning that Q_x must be ***positively charged****.*

We can consider breaking up the rod into small segments, each of equal charge, dQ. Each of these segments will exert a repulsive force on the +q charge. Due to the symmetry in the problem, the y-components of these forces will cancel, leaving only the summation of the x-components. The rod therefore exerts a force on the +q charge that is less than what would be exerted if all the rod's charge were concentrated at a single point that is a distance s from the +q charge.

As a result, the magnitude of the charge Q_x must be ***less than*** *the magnitude of charge Q_R, since it is concentrated at a single point and located the same distance from the +q charge as that of the rod.*

21. [5 pts] In Lab A2, a group places two charges 0.50 m apart and measures the force on one of the charges. They repeat the experiment three times and tabulate their data as shown. The students report the best estimate of the force as (4.0 ± 0.1) N.

Trial	Force (N)
1	3.52
2	3.46
3	3.68

Have the students made any errors? If yes, state so explicitly and describe what value they should report and why. If not, explain why their reported value is correct.

The average of the data can be found as follows:

$$avg = \frac{3.52\ \text{N} + 3.46\ \text{N} + 3.68\ \text{N}}{3} = 3.55\ \text{N}$$

And the standard deviation as:

$$\sigma = \sqrt{\frac{(3.52\ \text{N} - 3.55\ \text{N})^2 + (3.46\ \text{N} - 3.55\ \text{N})^2 + (3.68\ \text{N} - 3.55\ \text{N})^2}{2}} = 0.11\ \text{N}$$

If the students wanted to use two significant figures in the error (as calculated by the standard deviation), they could report the value as (3.55 ± 0.11) N *or if they wanted to use one significant figure in the error, they could report the value as* (3.6 ± 0.1) N.