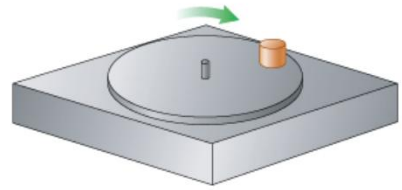
I. [45 points total] Lecture multiple-choice questions

1. [5 points] A cylinder sits on a circular turntable that is rotating at a constant speed as shown at right. The coefficient of static friction between the cylinder and the turntable is 0.080, and the cylinder is located 0.15 m from the center of the turntable. What is the maximum speed that the cylinder can move along its circular path without slipping off the turntable?



   A. 0.12 m/s
   B. 0.34 m/s
   C. 2.3 m/s
   D. 5.2 m/s
   E. We need to know the mass of the cylinder to answer.

2. [5 points] An ice skater is pulled by a horizontal rope in the negative $x$-direction across the ice, and there is negligible friction between the skates and the ice. Which of the following quantities could possibly be the work done by the rope on the skater, $W_{\mathrm{rope}}$, the displacement of the skater $\Delta\vec{x}$, and the tension exerted by the rope on the skater, $\vec{T}_{\mathrm{rs}}$, in this motion?
   A. $W_{\mathrm{rope}}$ = – 120 J, $\Delta\vec{x}$ = –2 m $\hat{\imath}$, $\vec{T}_{\mathrm{rs}}$ = + 60 N $\hat{\imath}$
   B. $W_{\mathrm{rope}}$ = + 120 J, $\Delta\vec{x}$ = –2 m $\hat{\imath}$, $\vec{T}_{\mathrm{rs}}$ = – 60 N $\hat{\imath}$
   C. $W_{\mathrm{rope}}$ = – 120 J, $\Delta\vec{x}$ = –2 m $\hat{\imath}$, $\vec{T}_{\mathrm{rs}}$ = – 60 N $\hat{\imath}$
   D. $W_{\mathrm{rope}}$ = + 120 J, $\Delta\vec{x}$ = +2 m $\hat{\imath}$, $\vec{T}_{\mathrm{rs}}$ = – 60 N $\hat{\imath}$
   E. $W_{\mathrm{rope}}$ = – 120 J, $\Delta\vec{x}$ = +2 m $\hat{\imath}$, $\vec{T}_{\mathrm{rs}}$ = + 60 N $\hat{\imath}$

3. [5 points] A small 1000 kg airplane's engine supplies 2.0 kN of thrust. Starting from rest, it reaches a take-off speed of 40 m/s in a distance of 600 m. What is the increase of thermal energy of the plane and runway system due to friction and drag?
   A. -400,000 J
   B. 0 J
   C. 400,000 J
   D. 800,000 J
   E. 1,200,000 J

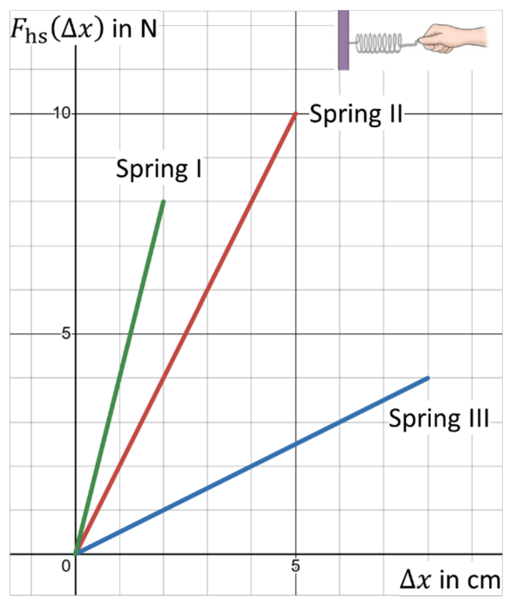
4. [5 points] A running coach is reviewing video from various races in order to make comparisons between her runners. She's compiled the data shown and would like to rank the runners based on the average power exerted in the given time intervals. Which of the following is the correct ranking from MOST to LEAST average power exerted?

| Runner | Mass (kg) | Δx (m) | initial speed (m/s) | final speed (m/s) | time interval (s) |
|---|---|---|---|---|---|
| *P* | 70 | 8.25 | 4 | 7 | 1.5 |
| *R* | 70 | 10.0 | 0 | 8 | 2.5 |
| *S* | 80 | 17.5 | 5 | 9 | 2.5 |

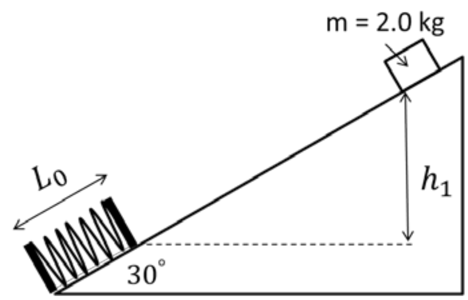
A. S > P > R
B. R = S > P
C. R > S > P
D. S > P = R
E. P > S = R

5. [5 points] Consider three different springs: I, II, and III. For each spring one end is attached to a wall, and you pull the other end to a maximum spring displacement. The graph shows your pulling force, $F_{\text{hs}}$, versus the spring displacement, $\Delta x$, for each spring. Rank the spring constants from the smallest to the largest.

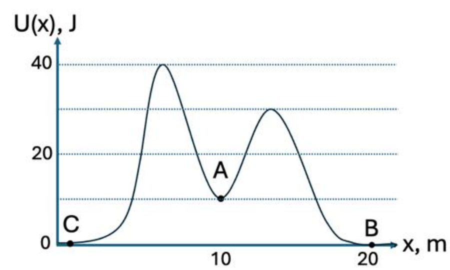
A. $k_{\text{III}} < k_{\text{II}} < k_{\text{I}}$
B. $k_{\text{I}} < k_{\text{II}} < k_{\text{III}}$
C. $k_{\text{II}} < k_{\text{I}} < k_{\text{III}}$
D. $k_{\text{III}} < k_{\text{I}} < k_{\text{II}}$
E. $k_{\text{I}} < k_{\text{III}} < k_{\text{II}}$



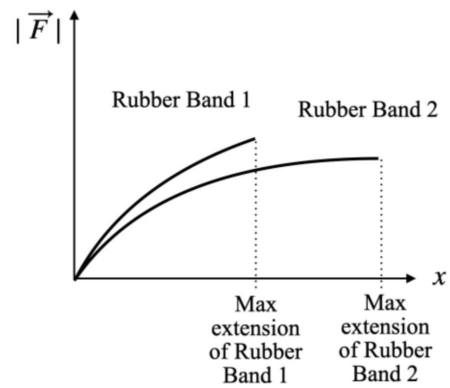
6. [5 points] A block of mass 2.0 kg is released from rest on a frictionless incline at a height $h_1$ of 1.0 m from the top of a spring. The incline angle is 30.0°. The spring is initially at its equilibrium length $L_0$ = 0.20 m and has a spring constant $k$ = 2000 N/m. At the instant when the spring is compressed by $d$ = 0.13 m, what is the speed of the block?
   A. 0 m/s
   B. 1.6 m/s
   C. 2.0 m/s
   D. 2.3 m/s
   E. Not enough information




7. [5 points] The graph at right shows the potential energy as a function of the position of a 1.0 kg particle. What is the speed needed for a particle starting at B to reach point A but not point C?
   A. 4.5 m/s < $v$, no maximum speed
   B. 7.7 m/s < $v$, no maximum speed
   C. 4.5 m/s < $v$ < 7.7 m/s
   D. 4.5 m/s < $v$ < 8.9 m/s
   E. 7.7 m/s < $v$ < 8.9 m/s



8. [5 points] Consider two rubber bands, 1 and 2. One end of each rubber band is held in place, and the other end is stretched from its equilibrium position by a hand, and the hand force necessary to stretch it is measured. There is no energy dissipation in the system.

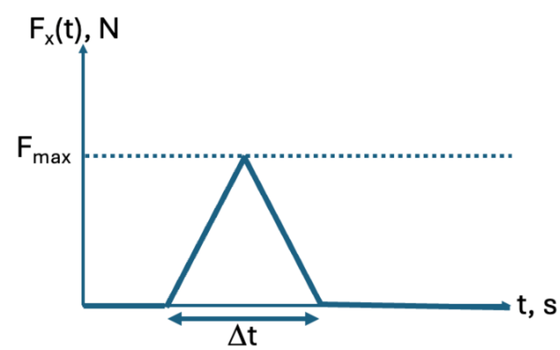
   A graph of the hand force as a function of the displacement of the rubber band ends is shown at right.

   How does the increase in potential energies of the two rubber bands compare when they are completely stretched?

   A. $\Delta U_1 < \Delta U_2$
   B. $\Delta U_1 > \Delta U_2$
   C. $\Delta U_1 = \Delta U_2$
   D. We need to know how the spring constants of the rubber bands compare.

9. [5 points] A ball of mass *m* hits a wall and rebounds with the same speed *v*. The figure below shows the force of the wall on the ball during the collision. What is the value of $F_{max}$?

   A. $\frac{1}{2}\frac{mv}{\Delta t}$
   B. $\frac{mv}{\Delta t}$
   C. $2\frac{mv}{\Delta t}$
   D. $4\frac{mv}{\Delta t}$
   E. $8\frac{mv}{\Delta t}$

II. [15 points total] Lab multiple-choice questions
Answer the following three questions based on the methods developed in the labs. For calculating uncertainty, use standard deviation.

10. [5 points] A group of students have measured the following four times for a coffee filter to fall through 1.5000 m. The students have also estimated an uncertainty of ± 0.0005 m in their distance measurement of 1.5000 m.

| Trial | Time (seconds) |
|---|---|
| 1 | 1.15 |
| 2 | 1.23 |
| 3 | 1.16 |
| 4 | 1.20 |

What is the fractional uncertainty in the speed?

A. 0.025
B. 0.028
C. 0.031
D. 0.035
E. 0.041

11. [5 points] A different group of students have determined that for a particular mass of their coffee filter, their average time is 1.21 s with a fractional uncertainty of 0.024. What was their standard deviation (in seconds)?

A. 0.02 s
B. 0.20 s
C. 0.03 s
D. 0.30 s
E. 0.06 s

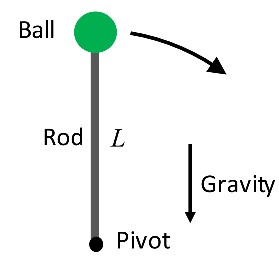
12. [5 points] Suppose you had to calculate a value $D$, where $D = E/F$. The value $E$ is given as 31.9 cm ± 4.1 cm, and the value $F$ is given as 9.52 s ± 0.12 s. Which one of the following is how you could report the value of D based on the guidelines introduced in the lab?

A. 3.35 cm/s ± 0.43 cm/s
B. 3.351 cm/s ± 0.431 cm/s
C. 3.4 cm/s ± 0.431 cm/s
D. 3.4 cm/s ± 0.2 cm/s
E. 3.35 cm/s ± 0.29 cm/s

III. [25 points total] Lecture free response questions
**<u>You must show your work to get the full credit.</u>**

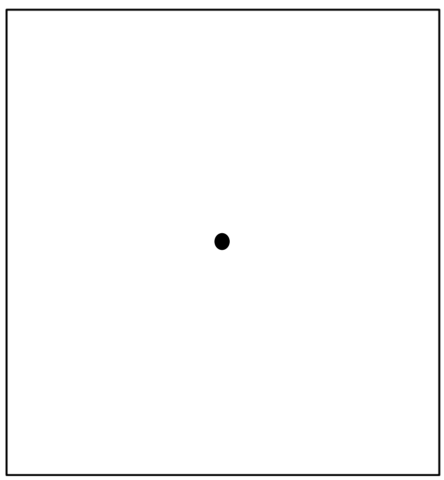
**Consider the following scenario for the next two questions.**
A small ball with mass $m$ is attached to one end of a rod. The other end of the rod is pivoted such that the rod rotates freely, and the ball undergoes circular motion. At the moment the ball is the top of the path as shown, the rod exerts a force on the ball with a magnitude $2mg$, where $g$ is the free-fall acceleration.

Side view




13. [4 points] In the box below, draw a free-body diagram for the ball at the moment shown. Make sure to label each force indicating the type of the force, the object exerting the force, and the object on which the force acts.

FBD of the ball



14. [5 points] What is the expression for the speed of the ball at the moment shown in terms of the variables given in the problem?

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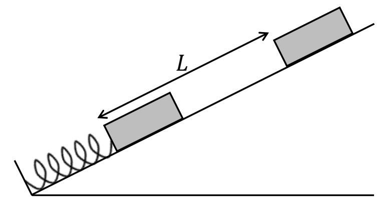
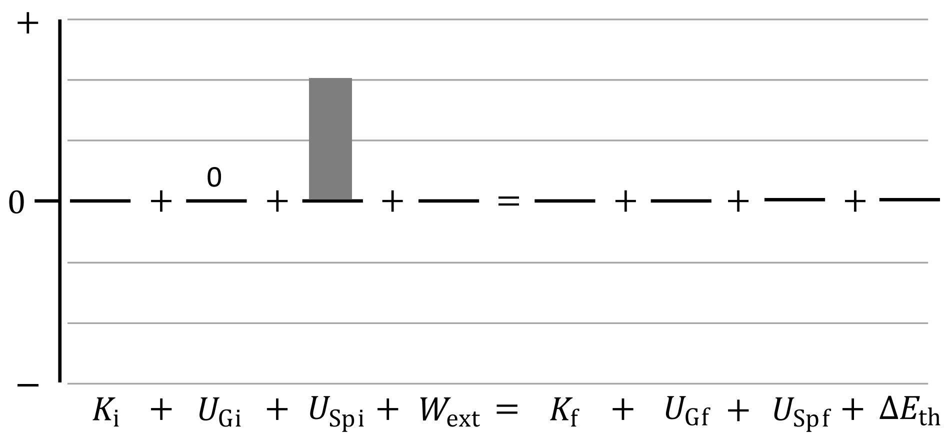
**Use the following scenario for the next two questions.**

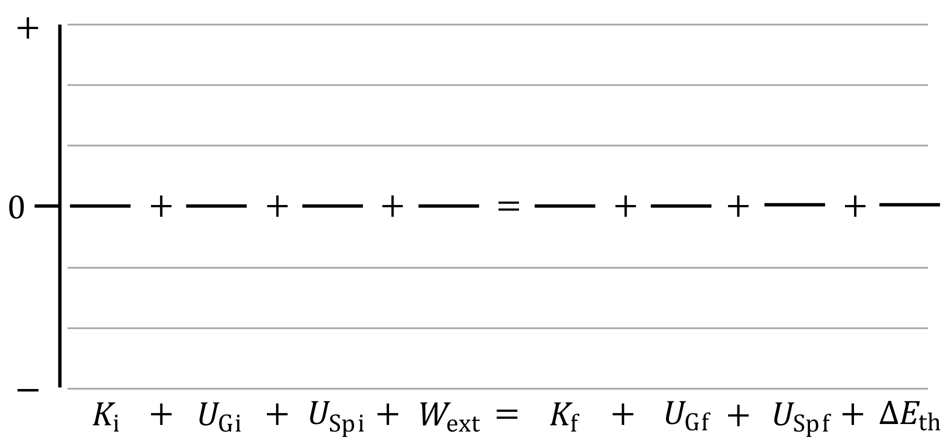
A tilted track has a spring at the base. A glider on the track slides with negligible friction. The glider is initially at rest and pushed against the spring compressing it by a distance $d$. Then the glider is let go and travels a distance $L$ up the incline when it reaches the maximum height. Note that $d \ll L$.



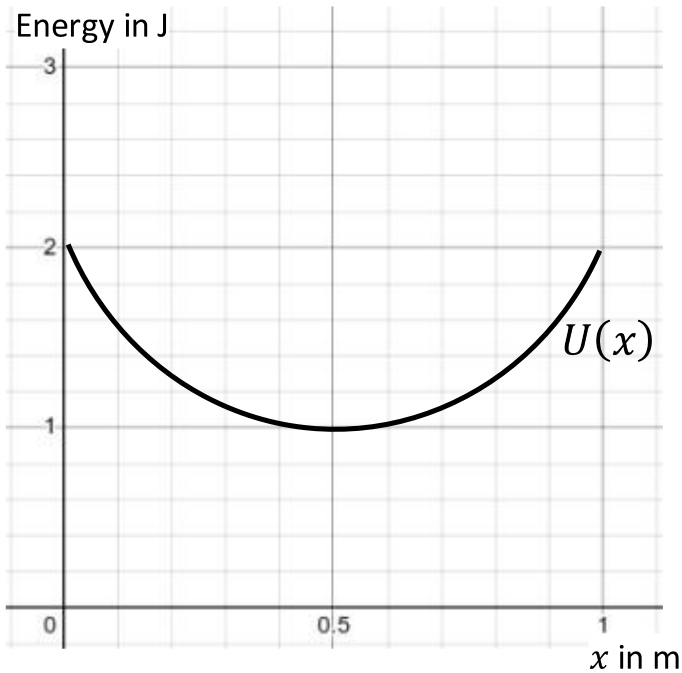

15. [6 points] Consider a system containing the glider, the spring, and the earth. The figure below is an incomplete energy bar chart for the system from the time the glider is released until it has traveled $\frac{L}{2}$ (halfway to the maximum height). Complete the chart. Make sure to draw it to the correct relative scale. If the quantity is zero, indicate so explicitly.




16. [6 points] Consider another system containing just the glider and the spring. Complete the chart for the same time interval. Make sure to draw it to the correct relative scale compared to the chart in Q15. If the quantity is zero or irrelevant, indicate so explicitly.



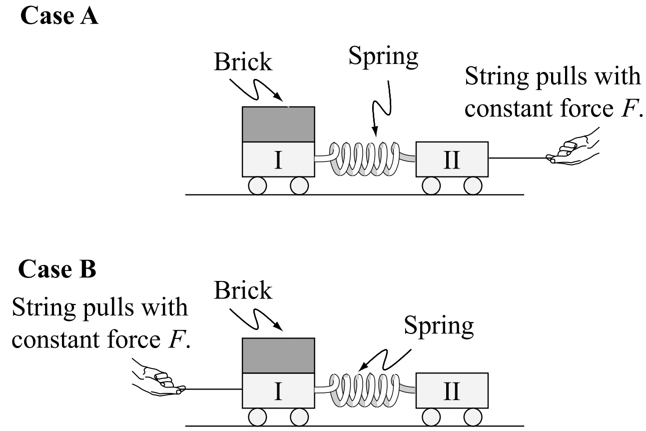
17. [4 points] A small particle near the bottom of a U-shaped track is sliding back and forth between $x = 0\ \mathrm{m}$ and $x = 1\ \mathrm{m}$ without any resistive force acting on it. The graph below shows the gravitational potential energy versus $x$ of the particle for a system that consists of the particle and the earth. In the graph below add the following graphs and label them clearly.
    A. Kinetic energy of the system versus $x$, $K(x)$
    B. Total mechanical energy of the system versus $x$, $E_{\mathrm{mech}}(x)$



IV. [15 points total] Tutorial free response questions (All three questions in this section are independent of one another.)

18. [5 points] Two identical carts are connected by a spring. On top of Cart I is a brick with mass equal to that of each cart.

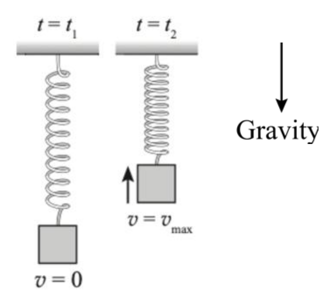
In case A, the carts are pulled by a string to the right. In case B, the string pulls to the left, as shown. In each case, the string exerts a constant force of magnitude $F$.



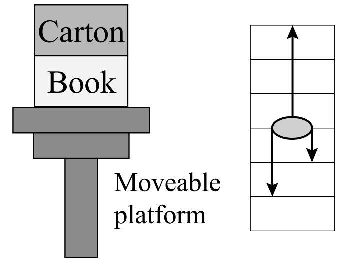

In which case is the spring length greater? Explain. (Assume the carts have been pulled for a long time so the spring is not oscillating. Ignore friction and the spring mass.)

19. [5 points] A block hangs from a spring as shown. At $t_1$ the string is stretched, and the block is at rest. At $t_2$ the block is moving upward. Let system $S_1$ consist of the block alone. Neglect air resistance.

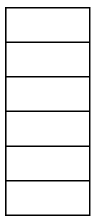
For the interval $\Delta t = t_2 - t_1$, is $W_{\text{net ext}}$, the net work done by external forces on system $S_1$ *positive, negative,* or *zero?* Explain.



20. [5 points] Two objects, a book and a carton are on a platform that can move upward or downward. The motion is not given, but the diagram at far right shows the relative magnitudes and directions of all the forces on one of the two objects (either the book or the carton).




In the space below, sketch a **possible** free-body diagram for the other object. (More than one diagram may be possible, but you need only sketch one.) **An explanation is not required** but draw your forces to the same scale as in the diagram above and label the forces to indicate the object exerting the force and the object on which the force is exerted.



**Constants**

| | |
|---|---|
| Free-fall acceleration | $g = 9.80\ \text{m/s}^2$ |
| Gravitational constant | $G = 6.67 \times 10^{-11}\text{N m}^2/\text{kg}^2$ |

**Mathematics**

| | |
|---|---|
| Vector components | $\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{\imath} + A_y\hat{\jmath}$ |
| Vector magnitude | $A = \sqrt{A_x^2 + A_y^2}$ |
| $\theta$ ccw from $x$-axis | $A_x = A\cos\theta$<br>$A_y = A\sin\theta$<br>$\theta = \tan^{-1}(A_y/A_x)$ |
| Adding vectors $\vec{C} = \vec{A} + \vec{B}$ | $C_x = A_x + B_x$<br>$C_y = A_y + B_y$ |
| Dot product | $\vec{A} \cdot \vec{B} = AB\cos\alpha$<br>$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y$ |
| Cross product | $\lvert\vec{A} \times \vec{B}\rvert = AB\sin\alpha$<br>$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$ |
| Standard deviation | $s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - x_{\text{ave}})^2}{n-1}}$ |

**Linear motion**

| | |
|---|---|
| Average velocity | $\vec{v}_{\text{ave}} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_{\text{f}} - \vec{r}_{\text{i}}}{t_{\text{f}} - t_{\text{i}}}$ |
| Instantaneous velocity | $\vec{v} = \frac{d\vec{r}}{dt}$ |
| Instantaneous acceleration | $\vec{a} = \frac{d\vec{v}}{dt}$ |

Constant acceleration (in "$s$" direction)

$$v_{\text{fs}} = v_{\text{is}} + a_s\Delta t$$
$$s_{\text{f}} = s_{\text{i}} + v_{\text{is}}\Delta t + \tfrac{1}{2}a_s(\Delta t)^2$$
$$v_{\text{fs}}^2 = v_{\text{is}}^2 + 2a_s\Delta s$$

| | |
|---|---|
| Motion on inclined plane | $a_s = \pm g\sin\theta$ |
| Relative motion | $\vec{v}_{\text{CB}} = \vec{v}_{\text{CA}} + \vec{v}_{\text{AB}}$ |

**Circular motion**

| | |
|---|---|
| Angular position | $\theta = \frac{s}{r}$ |
| Angular velocity | $\omega = \frac{d\theta}{dt}$ |
| Angular acceleration | $\alpha = \frac{d\omega}{dt}$ |
| Tangential velocity | $v_t = \omega r$ |
| Centripetal acceleration | $a_r = \frac{v_t^2}{r} = \omega^2 r$ |
| Tangential acceleration | $a_t = \alpha r$ |
| Period | $T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$ |
| Const. angular acceleration | $\omega_{\text{f}} = \omega_{\text{i}} + \alpha\Delta t$<br>$\theta_{\text{f}} = \theta_{\text{i}} + \omega_{\text{i}}\Delta t + \frac{1}{2}\alpha(\Delta t)^2$<br>$\omega_{\text{f}}^2 = \omega_{\text{i}}^2 + 2\alpha\Delta\theta$ |

**Force and motion**

| | |
|---|---|
| Newton's 2nd law | $\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$ |
| Gravity | $F_{\text{G}} = mg$<br>$F_{\text{G}} = \frac{GMm}{R^2}$ |
| Maximum static friction | $f_{\text{s max}} = \mu_{\text{s}}n$ |
| Kinetic friction | $f_{\text{k}} = \mu_{\text{k}}n$ |
| Reynolds number | $Re = \frac{\rho vL}{\eta}$ |
| Drag (high $Re$) | $F_{\text{drag}} = \frac{1}{2}C_{\text{d}}\rho Av^2$ |
| Drag (low $Re$) | $F_{\text{drag}} = 6\pi\eta rv$ |
| Newton's 3rd law | $\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$ |
| Circular motion | $(F_{\text{net}})_r = \frac{mv_t^2}{r} = m\omega^2 r$<br>$(F_{\text{net}})_t = ma_t$ |

**Work and energy**

| | |
|---|---|
| Kinetic energy | $K = \frac{1}{2}mv^2$ |
| Work by a constant force | $W = \vec{F} \cdot \Delta\vec{r}$ |
| Hooke's law | $(F_{\text{Sp}})_s = -k\Delta s$ |
| Work done by a spring | $W = -\frac{1}{2}k[(\Delta s_{\text{f}})^2 - (\Delta s_{\text{i}})^2]$ |
| Dissipative force | $\Delta E_{\text{th}} = f_{\text{k}}\Delta s$ |
| Potential energy | $\Delta U = -W_{\text{int}}$ |
| Grav. potential energy | $U_{\text{G}} = mgy$ |
| Elastic potential energy | $U_{\text{Sp}} = \frac{1}{2}k(\Delta s)^2$ |
| Mechanical energy | $\Delta E_{\text{mech}} = \Delta K + \Delta U$ |
| System energy | $\Delta E_{\text{sys}} = \Delta E_{\text{mech}} + \Delta E_{\text{th}}$<br>$\Delta E_{\text{sys}} = W_{\text{ext}}$ |

Power $P = \frac{dE_{\text{sys}}}{dt}$

$P = \vec{F} \cdot \vec{v}$

Force and potential energy $F_s = -\frac{dU}{ds}$

**Impulse and linear momentum**

Momentum $\vec{p} = m\vec{v}$

Impulse $\vec{J} \equiv \int_{t_\text{i}}^{t_\text{f}} \vec{F}(t)dt$

Momentum principle $\Delta\vec{p} = \vec{J}$

Isolated system $\vec{P}_\text{f} = \vec{P}_\text{i}$

Force $\vec{F} = \frac{d\vec{p}}{dt}$

Elastic collision with $(v_{\text{i}x})_2 = 0$

$$(v_{\text{f}x})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{\text{i}x})_1$$

$$(v_{\text{f}x})_2 = \frac{2m_1}{m_1 + m_2}(v_{\text{i}x})_1$$

**Rotation**

Center of mass $x_{\text{cm}} = \frac{1}{M}\sum_i m_i x_i$

Moment of inertia $I = \sum_i m_i r_i^2$

- Rod (center) $\frac{1}{12}ML^2$
- Rod (end) $\frac{1}{3}ML^2$
- Disk $\frac{1}{2}MR^2$
- Hoop $MR^2$
- Solid sphere $\frac{2}{5}MR^2$
- Hollow sphere $\frac{2}{3}MR^2$

Parallel-axis theorem $I = I_{\text{cm}} + Md^2$

Rotational kinetic energy $K_{\text{rot}} = \frac{1}{2}I\omega^2$

Torque $\tau \equiv rF\sin\phi = rF_\perp = Fd$

$\vec{\tau} \equiv \vec{r} \times \vec{F}$

Gravitational torque $\tau_{\text{grav}} = -Mgx_{\text{cm}}$

Newton's second law for rotation

$$\alpha = \frac{\tau_{\text{net}}}{I}$$

Critical angle $\theta_\text{c} = \tan^{-1}\left(\frac{t}{2\text{h}}\right)$

Rolling $v_{\text{cm}} = R\omega$

$K_{\text{rolling}} = \frac{1}{2}I_{\text{cm}}\omega^2 + \frac{1}{2}Mv_{\text{cm}}^2$

Angular momentum $\vec{L} \equiv \vec{r} \times \vec{p}$

Angular momentum of a particle

$$L_z = mrv_t = mr^2\omega$$

Angular momentum of a rigid body

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{\text{net}}$$

$$\vec{L} = I\vec{\omega}$$

**Theory of gravity**

Law of gravity $F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{Gm_1m_2}{r^2}$

Free-fall acceleration $g = \frac{GM}{r^2}$

Potential energy $U_\text{G} = -\frac{Gm_1m_2}{r}$

Satellite speed $v = \sqrt{\frac{GM}{r}}$

Kepler's third law $T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$

Escape speed $v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$

Geosynchronous orbit $r_{\text{geo}} = \left(\frac{GM}{4\pi^2}T^2\right)^{1/3}$