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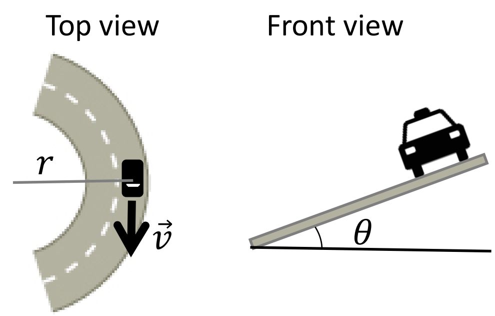
I. [45 pts] Multiple Choice (5 pts each): Mark your answer on BOTH the bubble sheet and this page.

1. [5 pts] You are designing a road that has a radius of curvature of r and is banked at an angle of θ. If cars of up to mass m are permitted on the highway, what is the maximum speed a car could go and still stay in its lane when it hits a patch of frictionless ice?



A. $\sqrt{rg\tan(\theta)}$

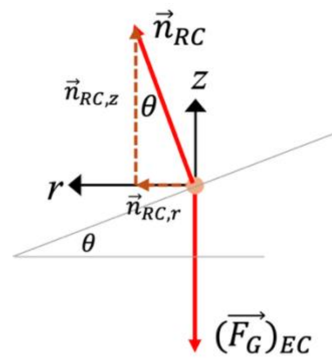
B. $\sqrt{rg\sin(\theta)}$

C. $\sqrt{rmg\cos(\theta)}$

D. $r\sqrt{mg\cos(\theta)}$

E. Cannot be determined from the information given

If the car hits a patch of frictionless ice, there are only two forces on the car, the normal force by the road and the gravitational force by the Earth. Only the normal force has a component that points along the radial axis (axis that points toward the center of the car's circular motion). The horizontal component of the normal is equal to the net radial force.



$$F_r = n_{RC,r} = m\frac{v^2}{r}$$

$$n_{RC}\sin\theta = m\frac{v^2}{r}$$

The vertical component of the normal force balances the gravitational force, as the car has no acceleration in the vertical direction (z axis).

$$n_{RC}\cos\theta = mg$$

$$n_{RC} = \frac{mg}{\cos\theta}$$

We can now combine these equations:

$$n_{RC}\sin\theta = \frac{mg}{\cos\theta}\sin\theta = m\frac{v^2}{r}$$

$$g\tan\theta = \frac{v^2}{r}$$

$$v = \sqrt{gr\tan\theta}$$

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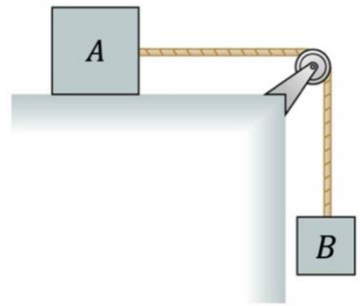
2. [5 pts] Block A, with mass m_A, is accelerated across a frictionless table by the falling block B, with mass m_B. The string is massless, and the pulley is both massless and frictionless. Defining the system as only block A, how much work does the tension force do during a time Δt that starts when the blocks are released from rest? (Assume that neither block hits the floor or the pulley during that time.)



A. 0

B. $\frac{m_A[g(\Delta t)]^2}{2}$

C. $\frac{1}{2m_A}[m_B g(\Delta t)]^2$

D. $\left(\frac{m_A m_B g}{m_A+m_B}\right)(\Delta t)$

E. $\frac{m_A}{2}\left(\frac{m_B g(\Delta t)}{m_A+m_B}\right)^2$

The work done by the tension force is equal to the change in kinetic energy of block A.

$$W_{tension} = \Delta K = \frac{1}{2}m_A\left(v_f^2 - v_i^2\right)$$

$$W_{tension} = \frac{1}{2}m_A\left(v_f^2 - 0^2\right) = \frac{1}{2}m_A v_f^2$$

We need a way to express the final speed of block A, v_f in terms of the time interval Δt. Since the initial speed of block A is zero, we can state that the final speed is: $v_f = a\Delta t$. Let us first look at block A – the net force on block A is equal to the tension force on block A as the normal force and gravitational force sum to zero.

$$F_{net,A} = T = m_A a$$

Now consider block B. There is an upward tension force and downward gravitational force. Setting the positive direction to be rightward for block A, means that the downward direction for block B is positive. We can write a Newton's second law equation for block B:

$$F_{net,B} = m_B g - T = m_B a$$

Since the two blocks are connected, the accelerations of each block are the same. We can combine the previous two equations to find the acceleration:

$$m_B g - T = m_B a$$

$$m_B g - (m_A a) = m_B a$$

$$m_B g = m_A a + m_B a$$

$$m_B g = a(m_A + m_B)$$

$$a = \frac{m_B g}{m_A + m_B}$$

The final speed of block A is thus $\frac{m_B g}{m_A+m_B}\Delta t$. *This can now be substituted into the work equation.*

$$W_{tension} = \frac{1}{2} m_A v_f^2 = \frac{m_A}{2}\left(\frac{m_B g \Delta t}{m_A + m_B}\right)^2$$

3. [5 pts] A 5.0-kg block initially at rest is pulled to the right along a horizontal surface by a constant horizontal force of 18 N. The coefficient of static friction between the block and the surface is 0.16 and the coefficient of kinetic friction between the block and the surface is 0.12. What is the change in thermal energy of the block-floor system after the block has moved 4.0 m?

   A. 24 J

   B. 31 J

   C. 41 J

   D. 48 J

   E. 72 J

The change in thermal energy of the block-floor system can be calculated as follows:

$$\Delta E_{th} = f_k \Delta s$$

The friction force is given as $\mu_k n$ *and since the applied force is horizontal, the normal force will be equal in magnitude to the gravitational force, mg.*

$$\Delta E_{th} = \mu_k n \Delta s = \mu_k m g \Delta s = (0.12)(5.0\text{ kg})(9.8\text{ m/s}^2)(4.0\text{ m}) = 24\text{ J}$$

4. [5 pts] An elevator has a mass of 2000 kg and is carrying passengers who have a combined mass of 300 kg. A constant friction force of 2100 N acts in opposition to the elevator's motion upward. If the upward force is provided by a motor, how much power delivered by the motor is required to lift the elevator at a constant speed of 2.00 m/s?

   A. 35,000 W

   B. 40,900 W

   C. 43,400 W

   D. 49,300 W

   E. Cannot be determined from the information given

   To move the elevator at constant speed, the force delivered by the motor must balance the two downward forces on the elevator, (1) the downward gravitational force and (2) the friction force.

$$\left|\vec{F}_{by\ motor\ on\ elevator}\right| = \left|\left(\vec{F}_G\right)_{Ee}\right| + \left|\vec{f}_{se}\right|$$

$$\left|\vec{F}_{by\ motor\ on\ elevator}\right| = m_{total}g + \left|\vec{f}_{se}\right|$$

$$\left|\vec{F}_{by\ motor\ on\ elevator}\right| = (2300\ \text{kg})(9.8\ \text{m/s}^2) + 2100\ \text{N} = 24{,}640\ \text{N}$$

   The power delivered by the motor can be determined as follows:

$$P = Fv = (24{,}640\ \text{N})(2\ \text{m/s}) = 49{,}300\ \text{W}$$

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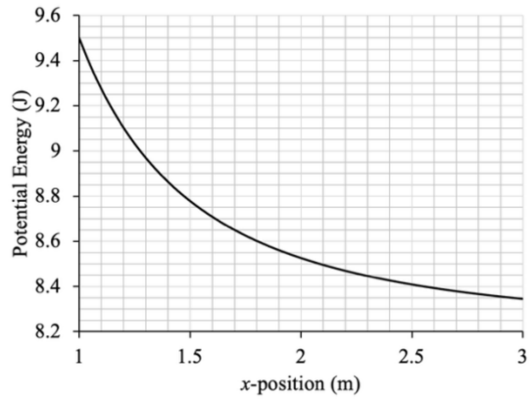
5. [5 pts] A particle is rolling around in a potential well given by $U_G(x) = 1.3x^{-2} + 8.2$, where U is in Joules and x is in meters. The function is also plotted at right for the range $x = 1$ m to $x = 3$ m. What is the force on the particle when it is at position $x = 2.1$ m?



A. -5.46 N

B. -0.28 N

C. 0.28 N

D. 7.7 N

E. 8.5 N

The relationship between the force and the potential energy versus position curve is:

$$F = -\frac{dU}{dx}$$

Using the power rule, the first derivative of the potential energy function is stated below:

$$\frac{dU}{dx} = -2.6x^{-3}$$

The force can now be determined:

$$F(2.1 \text{ m}) = -\frac{dU}{dx} = -(-2.6x^{-3})$$

$$F(2.1 \text{ m}) = 2.6x^{-3} = 2.6(2.1 \text{ m})^{-3} = 0.28 \text{ N}$$

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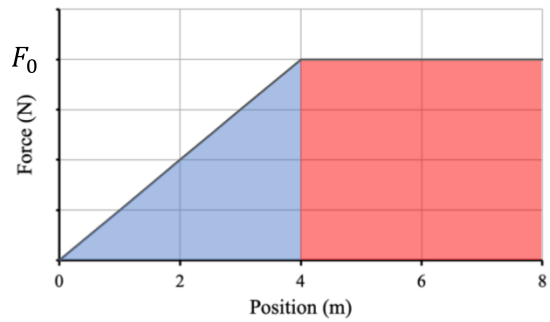
6. [5 pts] The graph at right shows the force applied to an object as the object moves from $x = 0$ m to $x = 8$ m. The force is applied parallel to its displacement and the object is at rest at $x = 0$ m and moves with a speed v at $x = 4$ m. What is the speed of the object at $x = 8$ m? Assume there are no other forces acting on the object.



A. $\frac{3}{2}v$

B. $\sqrt{2}v$

C. $\sqrt{3}v$

D. $2v$

E. $4v$

Since the system consists of a single object and there is only one force acting on the object, we can consider the net work done on the object as being equal to object's change in kinetic energy. And the work done by this force is equal to the area under the curve.

$$W_{net} = area = \Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

Let's say the max force is F_0 and determine the total area under the curve from x = 0 m to x = 8 m.

$$W = \frac{1}{2}(4\text{ m})(F_0) + (4\text{ m})(F_0) = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

$$(6\text{ m})(F_0) = \frac{1}{2}m\left(v_f^2 - 0^2\right)$$

$$(12\text{ m})(F_0) = mv_f^2$$

To make a connection to the speed v, we can consider the displacement from x = 0 m to x = 4 m, and determine the area under the curve.

$$W = \frac{1}{2}(4\text{ m})(F_0) = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

$$W = \frac{1}{2}(4\text{ m})(F_0) = \frac{1}{2}m(v^2 - 0^2)$$

$$(4\text{ m})(F_0) = mv^2$$

Let's return to the equation $(12\text{ m})(F_0) = mv_f^2$. *We can write it as:*

$$(12\text{ m})(F_0) = 3[(4\text{ m})(F_0)] = mv_f^2$$

$$(12\text{ m})(F_0) = 3[mv^2] = mv_f^2$$

$$v_f^2 = 3v^2$$

$$v_f = \sqrt{3}v$$

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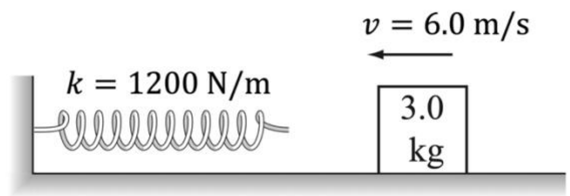
7. [5 pts] A block of mass 3.0 kg is moving across a horizontal, frictionless surface at a speed of 6.0 m/s. It then collides with a spring with spring constant $k = 1200$ N/m. Determine the maximum compression of the spring.



A. 0.12 m

B. 0.21 m

C. 0.30 m

D. 0.55 m

E. 0.81 m

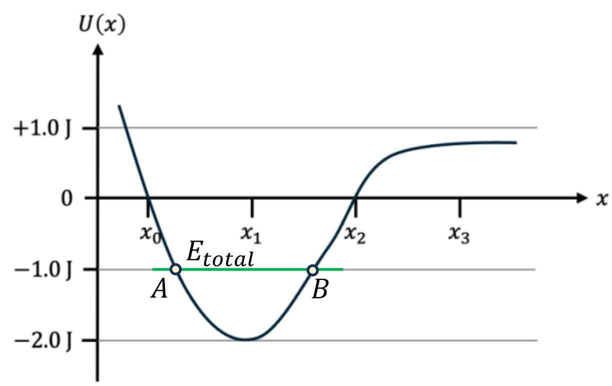
Let us consider a system of the block and spring. There is no external work done on the system, so the total energy of the system remains constant.

$$U_{Ei} + K_i = U_{Ef} + K_f$$

$$0 + \frac{1}{2}mv_i^2 = \frac{1}{2}k\Delta s^2 + 0$$

$$\Delta s = \sqrt{\frac{mv_i^2}{k}} = \sqrt{\frac{(3.0\text{ kg})(6\text{ m/s})^2}{1200\text{ N/m}}} = 0.3\text{ m}$$

8. [5 pts] A conservative force has the potential energy function $U(x)$, shown in the graph at right. A particle moving in one dimension under the influence of this force has 1.0 J of kinetic energy when it is at position x_1. Which of the following is a correct statement about the motion of the particle?



A. It oscillates with maximum position x_2 and minimum position x_0.

B. It moves to the right of x_3 and does not return.

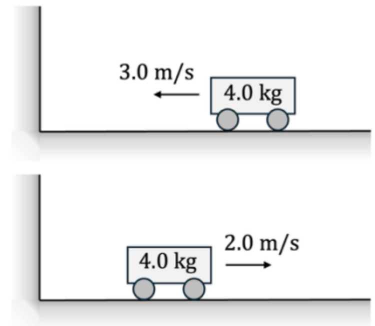
C. It moves to the left of x_0 and does not return.

D. It comes to rest at either x_0 or x_2.

E. It cannot reach either x_0 or x_2.

When the particle is at $x = x_1$, it has a kinetic energy of 1 J and the potential energy of the system is -2 J. This means that the total energy of the system is -1 J. This means that the object will oscillate between points A and B in the diagram above, which is consistent with the object not being able to reach either x_0 or x_2.

9. [5 pts] A 4.0-kg cart moves with a speed of 3.0 m/s along a frictionless track toward a wall. After colliding with the wall, the cart moves to the right with a speed of 2.0 m/s. The graph at right shows the force exerted on the cart while it is contact with the wall. How long is the cart in contact with the wall?



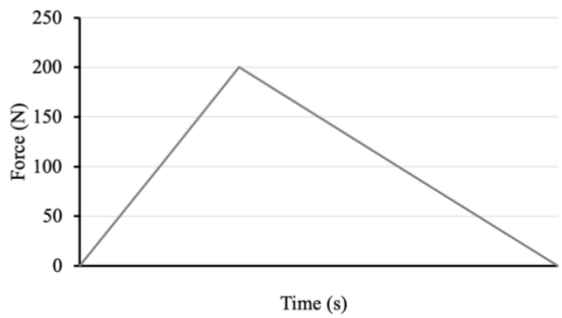
A. 0.02 s

B. 0.04 s

C. 0.1 s

D. 0.2 s

E. 0.4 s



The area under the curve of a force versus time is equal to the impulse imparted on an object, and the impulse is equal to the change in the object's momentum.

$$area = \vec{J} = \Delta\vec{p}$$

$$\frac{1}{2}\Delta t F_{max} = m(v_f - v_i)$$

$$\Delta t = \frac{2m(v_f - v_i)}{F_{max}} = \frac{2(4.0\text{ kg})\big((2.0\text{ m/s}) - (-3.0\text{ m/s})\big)}{200\text{ N}} = 0.2\text{ s}$$

II. [15 points total] Lab multiple-choice questions

The situation below applies to the following two questions.

10. [5 pts] A group of students measures the time it takes a coffee filter to fall 1.50 m. They drop the filter three times and calculate the **standard deviation** for the time measurements. (See table.)

Trial	Time (s)
1	1.89
2	2.01
3	1.82
Average time	1.90666
Standard deviation	0.09609

Which one of the following choices represents a correct reporting of the average time and its uncertainty according to the guidelines in the labs?

A. 1 ± 0.1 s

B. 2 ± 0.1 s

C. 1.9 ± 0.09 s

D. 1.9 ± 0.1 s

E. 1.91 ± 0.09 s

To one significant figure, the standard deviation is 0.1 s. Thus, the average should be reported to the tenths place, giving 1.9 ± 0.1 s. If one were to report the result to 2 significant figures, the standard deviation and average would be 1.91 ± 0.10 s. Only the former is shown (choice D).

11. [5 pts] The students calculate the velocity of the coffee filter from the previous question on a calculator and obtain 0.78671329 m/s. What is the uncertainty in the velocity to one significant figure? Assume that the uncertainty in the distance is small enough that it can be ignored.

    A. 0.1 m/s

    B. 0.04 m/s

    C. 0.05 m/s

    D. 0 m/s

    E. More information is needed.

*The fractional uncertainty in the time is the uncertainty divided by the average time (0.1 s/1.9 s) ~ 0.05 or about 5%. The fractional uncertainty in the velocity is the same as the largest fractional uncertainty in the distance or time. Since the uncertainty in the distance is small, the uncertainty in the velocity is 0.05 * 0.78671329 m/s = 0.04 m/s.*

12. [5 pts] A group of students conduct Lab A2 in which a ball rolls down a ramp. They release the ball from rest 10 times and record the time at which the ball reaches the mark that is 20 cm from the start.

    They calculate the following:

    - the average time,
    - the maximum deviation of the individual times from the average time, and
    - the standard deviation of the times.

    Which of the following would you expect to be true in general?

    A. The maximum deviation is ***greater than*** the standard deviation.

    B. The maximum deviation is ***equal to*** the standard deviation.

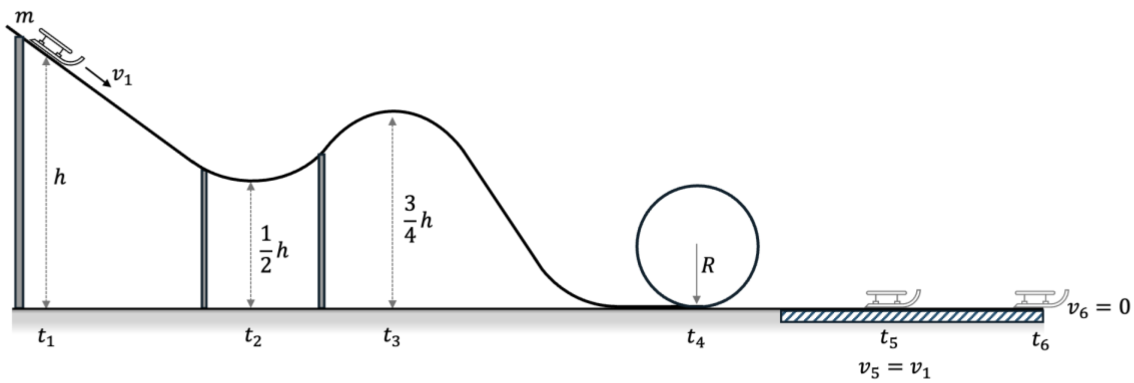
    C. The maximum deviation is ***less than*** the standard deviation.

    D. Not enough information to answer.

The maximum deviation is the largest difference between the average and the measured values. The standard deviation is a measure of the spread of the values around the mean. As found in lab, for a typical distribution, the maximum deviation is greater than the standard deviation.

III. Lecture long-answer questions (25 points total)

At t_1, a sled of mass m moves down a frictionless track with an initial speed v_1 as shown below. After descending through a height h, the sled encounters a loop-the-loop. It goes around once and then comes to rest after traversing a rough track that has a coefficient of friction μ_k. We will ask a series of questions about the motion of the sled. Times and other variables are shown in the sketch. Ignore air resistance.



13. [5 pts] In terms of the variables given in the problem and the free-fall acceleration g, what is the speed v_4 of the sled at time t_4, just before it encounters the loop-the-loop? Show your work.

Consider a system of the sled and Earth. There is no work done on this system. The initial gravitational potential energy and initial kinetic energy will be transformed into kinetic energy at t_4.

$$U_{G1} + K_1 = U_{G4} + K_4$$

$$mgh + \frac{1}{2}mv_1^2 = 0 + \frac{1}{2}mv_4^2$$

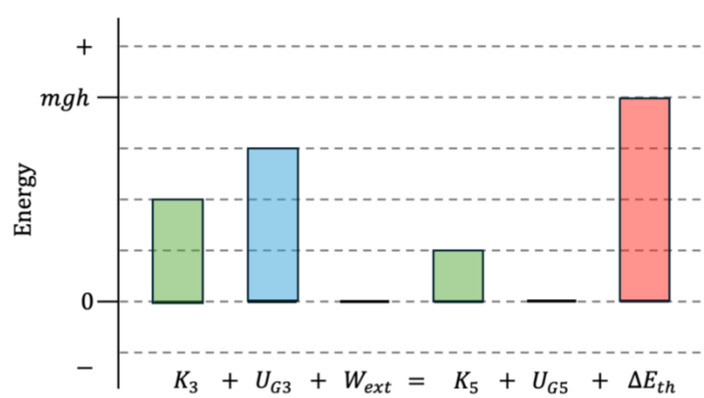
$$2gh + v_1^2 = v_4^2$$

$$v_4 = \sqrt{2gh + v_1^2}$$

14. [6 pts] Consider a system that includes the sled, the Earth and the track.

At $\boldsymbol{t_1}$, assume that the kinetic energy of the sled is **1/4th** of the potential energy of the system at this instant. The potential energy of the system at the ground level is zero.

Using the template at right, draw an Energy Bar Chart for the sled-Earth-track system for the time interval from $\boldsymbol{t_3}$ to $\boldsymbol{t_5}$.



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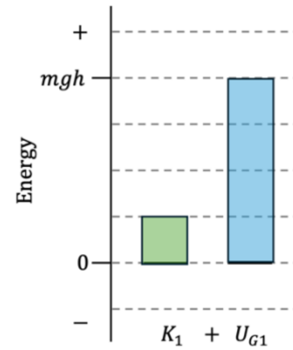
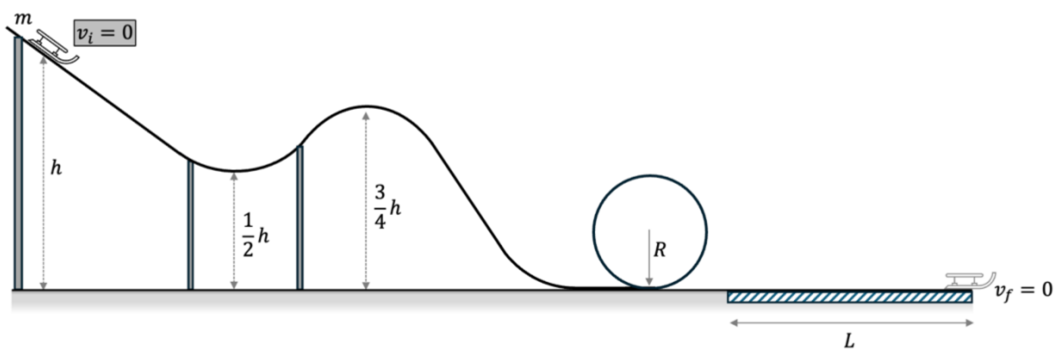
The question states that "At $\mathbf{t_1}$*, assume that the kinetic energy of the sled is **1/4th** of the potential energy of the system at this instant. The potential energy of the system at the ground level is zero". The energy graph at* t_1 *is shown at right. At* t_3*, the potential energy is* $\frac{3}{4}mgh$*, so three units on the energy graph above. Since the potential energy decreased by 1 unit from* t_1*, the kinetic energy increases from 1 unit to 2 units at* t_3*.*



There is no work done on the sled-Earth-track system.

At t_5*, the sled is moving at the same speed as at* t_1*, so there should be one unit of kinetic energy at* t_5*. The total energy of the system should remain constant as there is no work done. This means that the change in thermal energy should be 4 units.*



For the next three questions, suppose that the initial velocity (v_i) of the sled is zero as shown above.

15. [5 pts] After passing through the loop-the-loop, the sled slows and stops after traveling a distance L over a rough patch with coefficient of friction μ_k. What is the length L in terms of the other variables in the problem? Show your work.

Consider a system of the sled, Earth and track. The total energy of this system is constant.

$$U_{Gi} + K_i = U_{Gf} + K_f + \Delta E_{th}$$

$$mgh + 0 = 0 + f_k L$$

$$mgh = \mu_k n L$$

$$mgh = \mu_k mgL$$

$$L = \frac{h}{\mu_k}$$

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16. [4 pts] **Note that $\boldsymbol{v_i = 0}$**. The loop-the-loop has a radius R. What is the minimum speed the sled can have at the top of the loop, v_{top}, such that it makes it safely all the way around the loop? Express your answer in terms of the variables given in the problem and the free-fall acceleration g. Show your work.

    Hint: What is the minimum value of the normal force exerted on the sled when it is at the top of the loop?

    At the top of the loop, the minimum value of the normal force is zero. This means that the minimum radial force is provided alone by the gravitational force. The minimum speed can be written as follows:

$$F_{r,min} = m\frac{v_{min}^2}{R}$$

$$F_{r,min} = mg = m\frac{v_{min}^2}{R}$$

$$v_{min,top} = \sqrt{gR}$$

17. [5 pts] Noting that the sled is initially **at rest**, $v_i = 0$**,** what is the largest value of ***R*** (in terms of ***h*** and other variables) that would keep it on the track and allow it to make the entire loop-the-loop before exiting? Show your work.

    Consider a system of the sled, Earth and track. The total energy of this system is constant. Consider the initial energy when the sled is released from rest at the top of the track and the final energy when the sled is at the top of the loop. The initial potential energy at the beginning must equal the potential energy and the kinetic energy when the sled is at the top of the loop.

$$U_{Gi} + K_i = U_{Gf} + K_f$$

$$mgh + 0 = mg(2\text{R}) + \frac{1}{2}mv_{top}^2$$

$$mgh = mg(2\text{R}) + \frac{1}{2}m(gR)$$

$$h = (2\text{R}) + \frac{1}{2}R$$

$$R = \frac{2}{5}h$$

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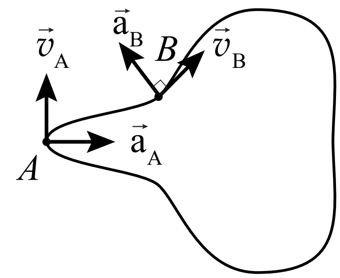
IV. Tutorial and lab long answer questions (15 points total)

18. [6 pts] A car moves clockwise at **constant** speed around the track shown in the top-view diagram at right. Two points, *A* and *B* on the track are marked.



a. [3 pts] Draw ***velocity*** vectors at each point. Draw them to make clear the relative magnitudes. State explicitly if the magnitudes are the same. Label them v_A and v_B.

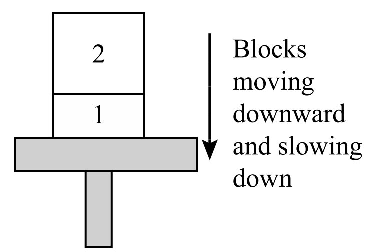
The velocity of an object is tangent to its path. And since the question states that the car is moving at constant speed, the velocity vectors should have equal magnitude.

b. [3 pts] Draw ***acceleration*** vectors at each point. Draw them to make clear the relative magnitudes. State explicitly if the magnitudes are the same. Label them a_A and a_B.

The car is moving at constant speed, so the car has only a radial acceleration. This means the acceleration should be directed perpendicular to the car's trajectory. The radial acceleration is inversely proportional to the radius of the turn ($a_r = v^2/r$). Since the radius of the turn at point A is greater than that at point B, the acceleration vector at point A should be larger in magnitude compared to that at point B.

Use the following scenario for the next two questions.

Two blocks, 1 and 2 are placed on a platform as shown at right. The mass of block 1 is less than that of block 2. The platform is moving downward and slowing down.



19. [4 pts] Is the magnitude of the **force by the platform on block 1** *greater than, less than,* or *equal to* the **force by block 2 on Block 1**? Explain.

The platform is slowing, so the acceleration and velocity have opposite directions. Thus, the acceleration is upward and so the net force is upward.

There are three forces on Block 1: $F_{by\ P\ on1}$ upward and $F_{by\ 2\ on\ 1}$ and $(F_G)_{by\ E\ on\ 1}$, both downward. Since the net force is upward, $F_{by\ P\ on1}$ is the largest and $\boldsymbol{F_{by\ P\ on\ 1} > F_{by\ 2\ on\ 1}}$.

20. [5 pts] While the platform is slowing down, is the total work done on **block 1** *positive, negative,* or *zero?* Explain.

*Since $K = \frac{1}{2}mv^2$ and the block is slowing, $\Delta K = K_f - K_i < 0$. There is no change in internal energy of the block, so the total work done by external forces is equal to the change in kinetic energy ($W_{total\ ext} = \Delta K$). Thus, $W_{total\ ext} < 0$ **(negative)**.*

Alternatively, the gravitational potential energy of block 1 is decreasing and the kinetic energy is decreasing so both are negative. $W = \Delta U + \Delta K$ so W is negative