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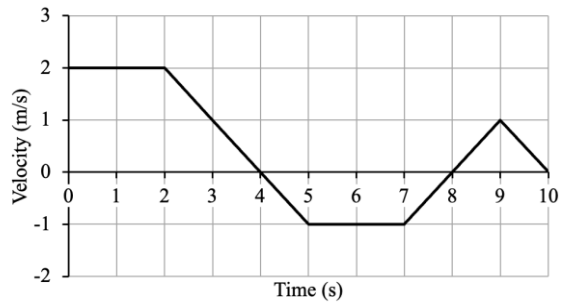
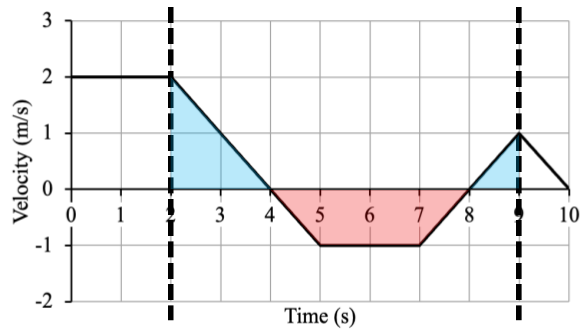
1. [5 pts] The figure shows the velocity-time graph of an object that is at $x = -3.0$ m at $t = 2.0$ s. What is the object's position at $t = 9.0$ s?



A. 10 m
B. 4.0 m
C. 1.0 m
D. -0.50 m
E. -3.5 m

At $t = 2.0$ s, the initial position x_0 is -3.0 m. The displacement position between x_0 and $t = 9.0$ s is Δx, the area under the velocity-time curve between those times. This is represented graphically at right, with positive displacements shown in blue in times when the velocity was in the positive direction and negative displacements, during times of negative-direction velocity, shown in red. The final position is therefore:



$$x_f = x_0 + \Delta x$$

$$x_f = -3.0\text{ m} + \frac{1}{2}\left(2\ \frac{\text{m}}{\text{s}}\right)(4\text{ s} - 2\text{ s}) - \frac{1}{2}\left(1\ \frac{\text{m}}{\text{s}}\right)(5\text{ s} - 4\text{ s}) - \left(1\ \frac{\text{m}}{\text{s}}\right)(7\text{ s} - 5\text{ s}) - \frac{1}{2}\left(1\ \frac{\text{m}}{\text{s}}\right)(8\text{ s} - 7\text{ s}) + \frac{1}{2}\left(1\ \frac{\text{m}}{\text{s}}\right)(9\text{ s} - 8\text{ s})$$

$$x_f = -3.5\text{ m}$$

This problem can be done with less arithmetic by crossing off equal areas in the positive-displacement and negative-displacement regions, then adding the remaining area (the net displacement) to the initial position:

$$x_f = x_0 + \Delta x = -3.0\text{ m} + (-0.5\text{ m}) = -3.5\text{ m}$$

2. [5 pts] A ball is tossed straight up from the ground with a speed of 16.0 m/s. When it returns, it falls into a hole 2.0 m deep. What is the ball's speed just before it hits the bottom of the hole?

A. 15 m/s
B. 17 m/s
C. 36 m/s
D. 48 m/s
E. 217 m/s

This is projectile motion purely in the y-direction, with no x-component to velocity or acceleration. Let's divide consider four moments: 1) the ball's launch from the ground 2) the moment it hits its highest point, 3) the moment it reaches the level of the ground again, and 4) the instant before it hits the bottom of the hole. The periods from 1->2 and 2->3 are symmetrical, with the same acceleration, the same magnitude of displacement, and each with zero velocity at the highest point. Illustrating this with one of the constant-acceleration kinematics equations, applied separately to these two time periods:

$$v_{2y}^2 = v_{1y}^2 + 2a_y\Delta y_{1\to2}$$

$$v_{2y}^2 = 0$$

$$\Rightarrow v_{1y}^2 = -2a_y\Delta y_{1\to2}$$

$$v_{3y}^2 = v_{2y}^2 + 2a_y\Delta y_{2\to3}$$

$$v_{2y}^2 = 0$$

$$\Rightarrow v_{3y}^2 = -2a_y\Delta y_{2\to3} = -2a_y\Delta y_{1\to2}$$

Therefore, v_{1y} *and* v_{3y} *have the same magnitude. That tells us that at time 3, the ball is traveling downward at 16.0 m/s. Now let's consider the time period 3->4. Applying the same constant-acceleration equation again to this time period:*

$$v_{4y}^2 = v_{3y}^2 + 2a_y\Delta y_{3\to4}$$

$$\Rightarrow\ v_{4y} = -\sqrt{v_{3y}^2 + 2a_y\Delta y_{3\to4}} = -\sqrt{\left(16.0\ \frac{\text{m}}{\text{s}}\right) + 2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(-2.0\ \text{m})} = -17\ \frac{\text{m}}{\text{s}}$$

The magnitude of this is $17\ \frac{m}{s}$.

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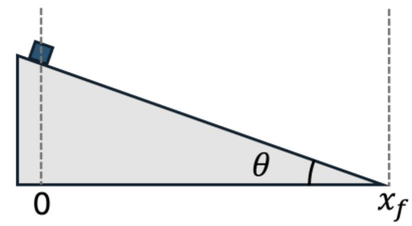
3. [5 pts] A block slides down a frictionless incline starting from rest as shown in the diagram. In terms of the horizontal location at the bottom of the ramp, x_f, and the angle of the incline, θ, what is the velocity of the block at the bottom of the slide?



A. $v_f = \sqrt{2gx_f \sin\theta}$
B. $v_f = \sqrt{2gx_f \cos\theta}$
C. $v_f = \sqrt{gx_f}$
D. $v_f = \sqrt{2gx_f \tan\theta}$
E. $v_f = \sqrt{gx_f \sin\theta}$

The block has an acceleration on the ramp that has a magnitude $g\sin\theta$. The distance the block moves along the track is $x_f / \cos\theta$. We can use constant acceleration kinematics to solve for the final speed (note the initial speed is zero).

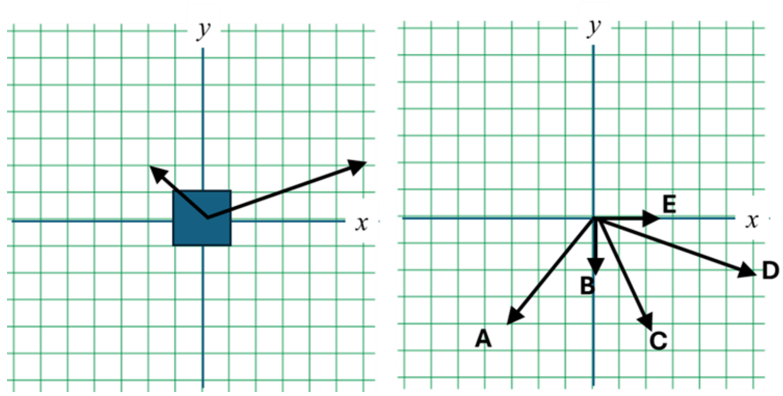
$$v_f^2 = v_i^2 + 2a\Delta s$$

$$v_f^2 = 2(g\sin\theta)\frac{x_f}{\cos\theta} = 2gx_f\tan\theta$$

$$v_f = \sqrt{2gx_f\tan\theta}$$

4. [5 pts] The diagram on the left shows a top-view diagram of a block sitting at the origin of an ice sheet (no friction). The two vectors represent two forces acting on the block.

The diagram on the right shows 5 additional vectors representing forces. Which one, when applied to the block, will result in an acceleration directly to the right along the x axis? All forces are in the xy plane.



Top-view diagram

A. Vector A
B. Vector B
C. Vector C
D. Vector D
E. Vector E

The sum of the y-components of two forces on the block in the leftmost diagram is 4 units in the positive y-direction. And the sum of the x-components is 4 units to the right. To have the net force point directly to the right, the third force must have a y-component with 4 units in the negative y-direction and must have an x-component that is either positive, or if negative, must have a magnitude less than four units. This is true for vectors A and C.

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5. [5 pts] A car of mass m is traveling on a flat circular track with diameter d. At what speed v_t would the car need to go for the magnitude of the force causing its centripetal acceleration to be equal to the magnitude of the gravitational force acting on it?

A. mgd
B. $\sqrt{gd}$
C. $\sqrt{\frac{mgd}{2}}$
D. $\sqrt{\frac{2g}{d}}$
E. $\sqrt{\frac{gd}{2}}$

To go around a circular track at a uniform speed, its centripetal acceleration must have a magnitude of: $a_c = \frac{v_t^2}{r}$, *and* $r = \frac{d}{2}$, *so* $a_c = \frac{2v_t^2}{d}$.

We know that $F_{net} = ma$.

Combining these, the force causing the centripetal motion has magnitude: $ma_c = \frac{2mv_t^2}{d}$.

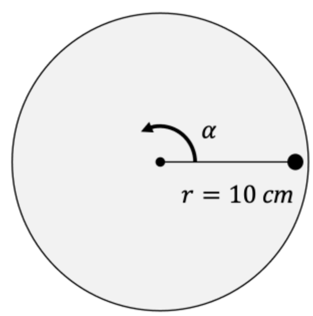
We want this to be equal in magnitude to the gravitational force, given by $F_G = mg$.

That means that: $mg = \frac{2mv_t^2}{d}$.

Cancelling out mass from both sides of the equation and solving for v_t, *we get*

$$v_t = \sqrt{\frac{gd}{2}}$$

6. [5 pts] A dot is drawn on a disc 10 cm from its center. The disc starts at rest at $t = 0$ s, then undergoes a constant angular acceleration of $\alpha = 3.0$ radians/s^2. What is the magnitude of the dot's total linear acceleration at $t = 0.71$ s?

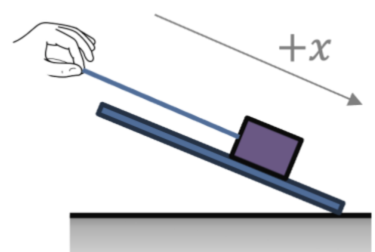


A. 0.30 m/s^2
B. 0.45 m/s^2
C. 0.54 m/s^2
D. 0.75 m/s^2
E. 45 m/s^2

The disc is undergoing constant angular acceleration, so $\omega_f = \omega_i + \alpha\,\Delta t$.
The disc begins at rest, so $\omega_i = 0$ *and therefore* $\omega_f = \alpha\,\Delta t$
The final centripetal acceleration is related to the final angular velocity by $a_{c,f} = \omega_f^2 r$.
Combining these, we get $a_{c,f} = \alpha^2(\Delta t)^2 r$.
We also know that the tangential acceleration is related to the angular acceleration by $a_{t,f} = \alpha r$.
The magnitude of the total acceleration is given by:

$$
\begin{aligned}
a_{tot} &= \sqrt{\left(a_{c,f}\right)^2 + \left(a_{t,f}\right)^2} \\
&= \sqrt{(\alpha^2(\Delta t)^2 r)^2 + (\alpha r)^2} \\
&= \sqrt{\left(\left(3.0\frac{radians}{s^2}\right)^2 (0.71\,s)^2 (0.1\,m^2)\right)^2 + \left(\left(3.0\frac{radians}{s^2}\right)(0.1\,m^2)\right)^2} \\
&= 0.54\ \mathrm{m/s^2}
\end{aligned}
$$

7. [5 pts] A block is moving down a ramp in the $+x$ direction. There is friction between the block and the surface, and a rope exerts a tension force pulling the block in the $-x$ direction. The block is slowing down. Which of the following is true about the forces on the block?



A. The net force on the block points in the $+x$ direction.
B. Static friction is exerting a force in the $+x$ direction.
C. Static friction is exerting a force in the $-x$ direction.
D. The force of kinetic friction is in the same direction as the tension force from the rope.
E. The component of gravity pointing in the $+x$ direction must be smaller in magnitude than the tension from the rope.

The block is moving down the ramp but its velocity is decreasing, so its acceleration is in the direction up the ramp, in the -x direction. This rules out A.

The block is moving with respect to the ramp, so the friction is kinetic, not static, ruling out B and C.

The kinetic friction opposes the motion of the block with respect to the ramp, so it points in the opposite direction to the block's velocity. The kinetic friction therefore points in the -x direction, the same direction as the tension force. Choice D is therefore correct.

Tension and kinetic friction, both pointing in the -x direction, must sum to a greater magnitude than the component of gravitational force in the +x direction in order to cause the acceleration in the -x direction. But neither of them needs to be larger on their own. Therefore, E is not true.

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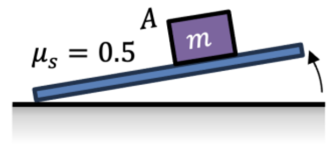
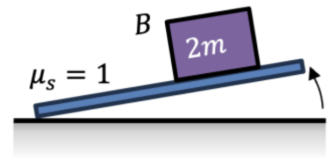
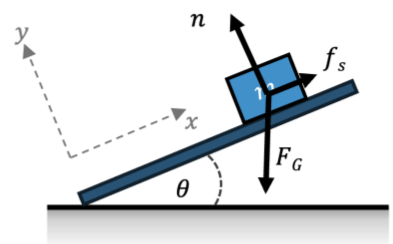
8. [5 pts] Block A of mass m is stationary on a ramp. The coefficient of static friction between block A and the ramp is 0.5. The angle of the ramp is increased until block A slips at angle θ_A. Block B, which has mass $2m$ and is made of a different material, is initially stationary on a different ramp. The coefficient of static friction between block B and its ramp is 1. The angle of the ramp is increased until block B slips at angle θ_B. How do θ_A and θ_B compare?



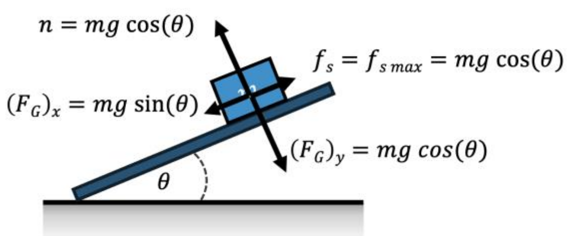


A. $\theta_B > 2\theta_A$
B. $2\theta_A > \theta_B > \theta_A$
C. $\theta_B = \theta_A$
D. $\theta_A > \theta_B$
E. Cannot be determined from the information given

For either case, we can first draw a free-body diagram and define axes:



We decompose the gravitational force along these axes. Because the block is not accelerating in the y direction, the normal force must be equal and opposite to the y component of the gravitational force.



In the x direction, the block starts to slip when static friction reaches its maximum:
$f_{s\,max} = n\mu_s = \mu_s mg\cos(\theta)$

And the x component of gravity becomes strong enough to overcome it:

$mg\sin(\theta) = \mu_s mg\cos(\theta)$

$\mu_s = \tan(\theta)$

Therefore, $\theta = \tan^{-1}(\mu_s)$.

For block A, with $\mu_s = 0.5$, $\theta_A = 27°$.

For block B, with $\mu_s = 1$, $\theta_B = 45°$.

$Therefore, 2\theta_A > \theta_B > \theta_A.$

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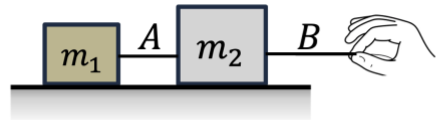
9. [5 pts] Two blocks are connected by rope A and pulled to the right on a horizontal frictionless table by rope B. If the tension in rope B is 3.0 N, $m_1 = 2$ kg and $m_2 = 7$ kg, what is the tension in the rope A?



A. 0.66 N
B. 0.85 N
C. 2.3 N
D. 3.0 N
E. 10.5 N

There is an acceleration constraint here; $a_1 = a_2$. We'll just call this a.
Let's first consider the two blocks and the rope between them as a system. It must be that $F_{net,system\ of\ 1\ and\ 2} = (m_1 + m_2)a$, and the only external force in the horizontal direction is $T_{B\ on\ 2}$.
Therefore, $a = \frac{T_{B\ on\ 2}}{m_1+m_2}$.
Now let's consider the system of just block 1. The tension of rope A is the only force acting on it in the horizontal direction, so
$T_{A\ on\ 1} = F_{net,1} = m_1 a$.
Substituting in our result for A above,

$$T_{A\ on\ 1} = m_1 \frac{T_{B\ on\ 2}}{m_1 + m_2} = (2\ kg)\frac{3.0\ N}{(2\ kg + 7\ kg)} = 0.66\ N$$

Lab Multiple Choice Questions

10. [5 pts] A group of students is using a digital stopwatch to make time measurements as part of a lab experiment. They perform three trials, and their data is shown at right. The students use the maximum deviation method to determine the random uncertainty in their measurements. For this case, how much larger is the random uncertainty compared to the instrumental uncertainty of the stopwatch?

Trial	Time
1	3.15 s
2	3.58 s
3	2.96 s

A. Random uncertainty is 16 times larger than the instrumental uncertainty.
B. Random uncertainty is 30 times larger than the instrumental uncertainty.
C. Random uncertainty is 50 times larger than the instrumental uncertainty.
D. Random uncertainty is 70 times larger than the instrumental uncertainty.
E. Random uncertainty is 270 times larger than the instrumental uncertainty.

For digital devices, the instrumental uncertainty is equal to half the smallest possible measurement. From the data above, the stopwatch can read to 1/100 of a second (0.01 s). The instrumental uncertainty of the stopwatch is therefore 1/200 of a second, or ***0.005 seconds****.*

To find the random uncertainty, we need to consider the maximum deviation of each measurement from the average of the three measurements. The calculations are shown below:

$$average = \frac{3.15\ \text{s} + 3.58\ \text{s} + 2.96\ \text{s}}{3} = 3.23\ \text{s}$$

Deviation from average

$$|3.15\text{ s} - 3.23\text{ s}| = 0.08\text{ s}$$

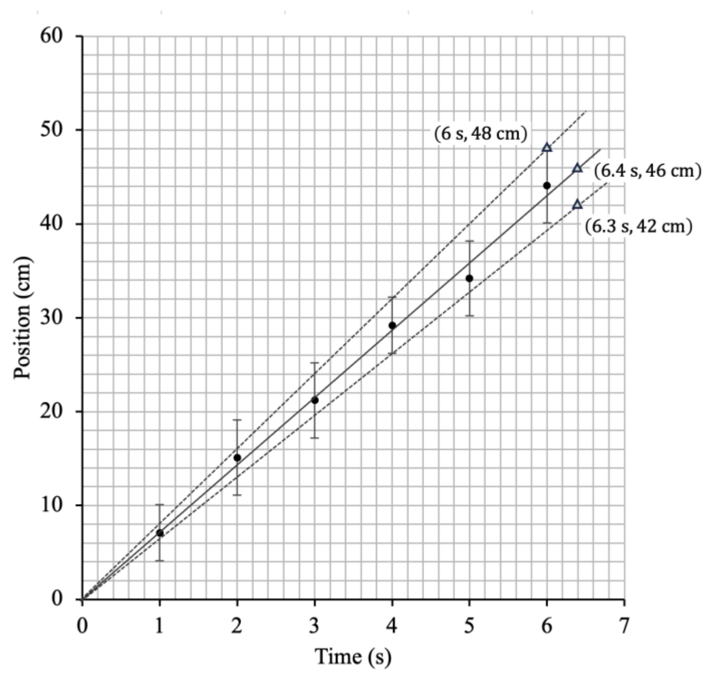
$$|3.58\text{ s} - 3.23\text{ s}| = 0.35\text{ s}$$

$$|2.96\text{ s} - 3.23\text{ s}| = 0.27\text{ s}$$

The maximum deviation is 0.35 s, which is 70 times larger than 0.005 s.

$$\frac{0.35\text{ s}}{0.005\text{ s}} = 70$$

11. [5 pts] A group of students (group A) have studied the motion of a toy car and formed the position-time graph at right. They have also plotted a best-fit line and lines of maximum and minimum slope. Coordinates on these lines (indicated by triangular data markers) are shown, and all three lines pass through the origin. Based on these values and the guidelines introduced in Labs A1/A2, which of the choices below represents a correct reporting of the velocity of the car?



A. $(6.3 \pm 1.47)\ cm/s$
B. $(6.3 \pm 1.5)\ cm/s$
C. $(7.19 \pm 0.81)\ cm/s$
D. $(7.2 \pm 0.6)\ cm/s$
E. $(7.2 \pm 1.1)\ cm/s$

We first need to find the slopes of the three lines.

$$m_{max} = \frac{48\text{ cm}}{6\text{ s}} = 8\text{ cm/s}$$

$$m_{best-fit} = \frac{46\text{ cm}}{6.4\text{ s}} = 7.188\text{ cm/s}$$

$$m_{min} = \frac{42\text{ cm}}{6.3\text{ s}} = 6.67\text{ cm/s}$$

We now need to find the deviation of the maximum slope from the best-fit slope, and the deviation of the minimum slope from the best-fit slope.

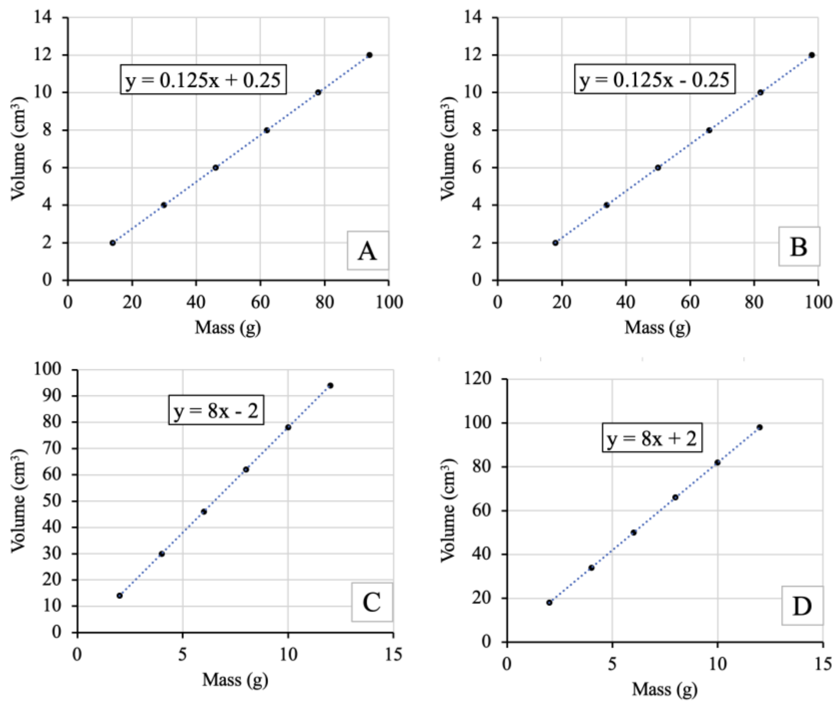
$$|m_{best-fit} - m_{max}| = |7.188\text{ cm/s} - 8\text{ cm/s}| = 0.813\text{ cm/s}$$

$$|m_{best-fit} - m_{min}| = |7.188\text{ cm/s} - 6.67\text{ cm/s}| = 0.52\text{ cm/s}$$

The maximum deviation is 0.81 cm/s. The uncertainty should have no more than two significant figures, and the last decimal place of the best estimate should be in the same place as the last decimal place in the uncertainty. We could thus report the velocity as $(7.2 \pm 0.8)\ cm/s$ *or* $(7.19 \pm 0.81)\ cm/s$*. Only the latter value is given in the choices.*

12. [5 pts] Students are given 6 different steel blocks. The density of steel is 8.00 g/cm^3 ($\rho = m/V$). The students use a mass scale to measure the mass of each block, and they use a graduated cylinder to measure the volume. They form a plot of volume versus mass.

After they make their plot, they notice that the mass scale reads -2.00 g when no object is on the scale. Which of the graphs at right would be closest to the student's volume versus mass plot?



A. Graph A
B. Graph B
C. Graph C
D. Graph D

We can start by rearranging the equation for density such that the volume is the subject of the equation:

$$V = \frac{1}{\rho} m$$

This is a linear equation ($y = mx$). So, if volume is plotted on the vertical axis and mass is plotted on the horizontal axis, then the slope is equal to $1/\rho$. Since the density is given as 8.00 g/cm^3, the slope of their graph should be 1/(8.00 g/cm^3) or 0.125 cm^3/g. Graphs C and D have the incorrect slopes so these choices cannot be correct.
The difference between graph A and B is the sign of the y-intercept.

From the information in the question, we know that $m_{scale} = m_{true} - 2.00\text{ g}$, which means that $m_{true} = m_{scale} + 2.00\text{ g}$. Let's substitute this equation into the previous equation:

$$V = \frac{1}{\rho} m_{true} = \frac{1}{\rho}(m_{scale} + 2.00\text{ g})$$

$$V = \frac{1}{\rho} m_{scale} + \frac{2.00\text{ g}}{\rho} = \left(0.125\ \frac{\text{cm}^3}{g}\right) m_{scale} + 0.25\text{ cm}^3$$

From these calculations, we can see that graph A has the correct slope and the correct intercept.

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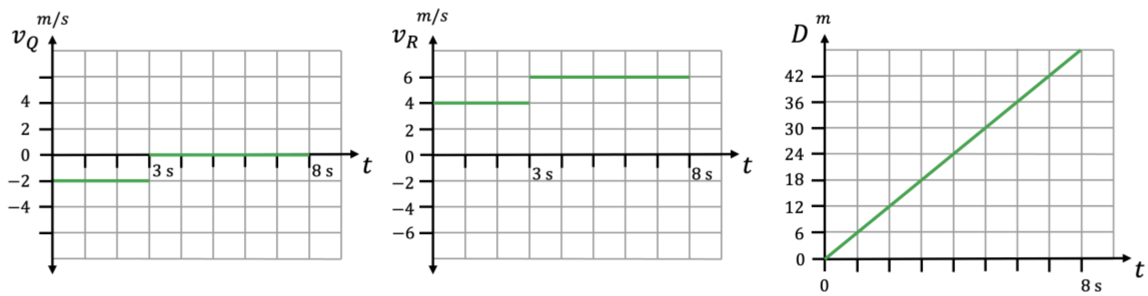
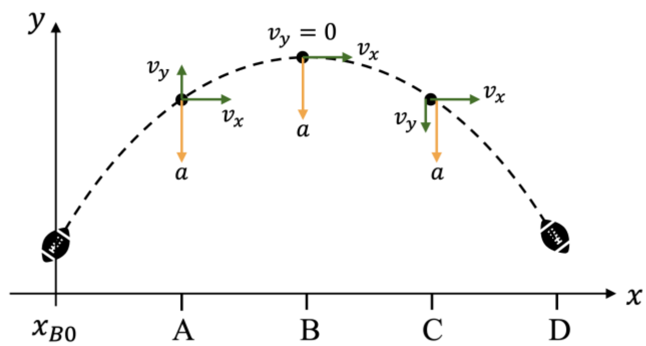
Lecture Free Response

13. [5 pts] A Quarterback (Q) and a Receiver (R) stand side by side at position $x = 0$. At $t = 0$, Q begins to run backwards with velocity $\boldsymbol{v_Q} = \mathbf{-2\ m/s}$ for 3 seconds, then Q stops. At the same time, R runs forward with velocity $\boldsymbol{v_R} = \mathbf{+4\ m/s}$ for 3 seconds and then instantaneously begins to run with $\boldsymbol{v_R} = \mathbf{+6\ m/s}$ for the next 5 seconds. On the axes below, draw a graph of v_Q and v_R vs time, and the total distance $\boldsymbol{D}$ between Q and R vs time. You can assume the time axis ticks are 1 second apart, but you must indicate the scale of your vertical axes (values and units).



14. [6 pts] When the quarterback stops, he launches the ball from location $\boldsymbol{x_{B0}}$ with velocity $\boldsymbol{v_{B0}}$ at an angle $\boldsymbol{\theta}$ from the horizontal. The trajectory of the ball is shown in the figure. Draw acceleration vectors (labeled a) and velocity vector <u>components</u> (labeled v_x and v_y) on the ball's trajectory at positions A, B, and C.



15. [5pts] <u>For this question, $t = 0$ is when the ball is thrown, and the initial vertical position of the ball is zero.</u> Recall that Q launches the ball from location $\boldsymbol{x_{B0}}$ with velocity $\boldsymbol{v_{B0}}$ at an angle $\boldsymbol{\theta}$ from the horizontal. At this instant, the receiver (R) is at x_{R0} and is moving with a speed v_R. In terms of the given variables, write equations that represent the $\boldsymbol{x}$ and $\boldsymbol{y}$ locations of the ball (B) and the $\boldsymbol{x}$ position of the receiver (R) as a function of time. Your final expressions should not include any terms which are zero.

Eqn. 1: $x_B(t) = x_{B0} + v_{BO}\cos\theta\, t$

Eqn. 2: $y_B(t) = v_{BO}\sin\theta\, t - \frac{1}{2}gt^2$

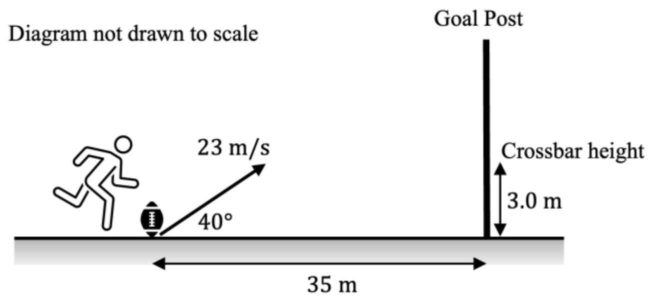
Eqn. 3: $x_R(t) = x_{RO} + v_R t$

16. [3 pts] The information above is insufficient to determine if the ball reaches the receiver or not. What conditions must be met for this to be a catch? You should not solve this, just state what is the relationship between the above expressions that represents a catch and explain your reasoning.

For the receiver to catch the ball, the vertical position of the ball must be the same as the receiver's vertical position, which in this case is zero. And the horizontal position of the ball must be the same as the horizontal position of the receiver.

So, for some time t, $x_B(t) = x_R(t)$ *and* $y_B(t) = 0$.

17. [6 pts] Later in the game, a field-goal kicker kicks a ball with a speed of 23 m/s at an angle of 40° from the horizontal. The goal post is 35 meters away and the goal post crossbar is 3.0 m above the ground. Assuming the aim is correct, how high is the ball above the ground when it passes the goal post? Show your work.



The question is seeking the vertical position of the ball when it has reached the goal post. We can write the vertical position of the ball as:

$$y(t) = v_{i,y}t - \frac{1}{2}gt^2 = \left((23\text{ m/s})\sin 40°\right)t - \frac{1}{2}gt^2$$

The above equation has two unknowns, since we also do not know the time at which the ball reaches the goal. But we can use the horizontal speed and horizontal distance to find the time.

$$x(t) = v_x t$$

$$t = \frac{x(t)}{v_i \cos 40°}$$

In this case, we are looking for the time at which the ball has traveled a horizontal distance of 35 m, so:

$$t = \frac{35\text{ m}}{(23\text{ m/s})\cos 40°}$$

We can now substitute this equation for time into the vertical position equation:

$$y(t) = \left((23\text{ m/s})\sin 40°\right)\left(\frac{35\text{ m}}{(23\text{ m/s})\cos 40°}\right) - \frac{1}{2}g\left(\frac{35\text{ m}}{(23\text{ m/s})\cos 40°}\right)^2$$

$$y(t) = 35\text{ m}(\tan 40°) - \frac{1}{2}g\left(\frac{35\text{ m}}{\left(23\frac{\text{m}}{\text{s}}\right)\cos 40°}\right)^2 = 10\text{ m}$$

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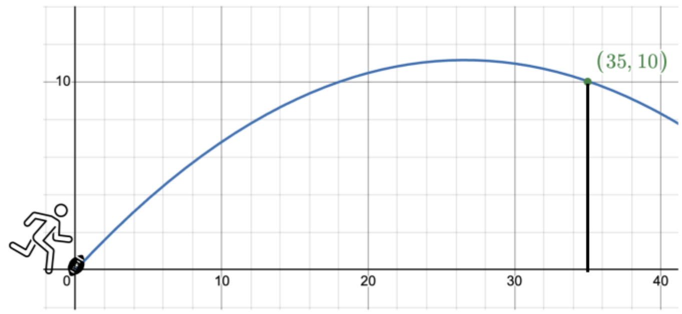
The diagram at right shows the trajectory of the football.



https://www.desmos.com/calculator/pk8wqluukb

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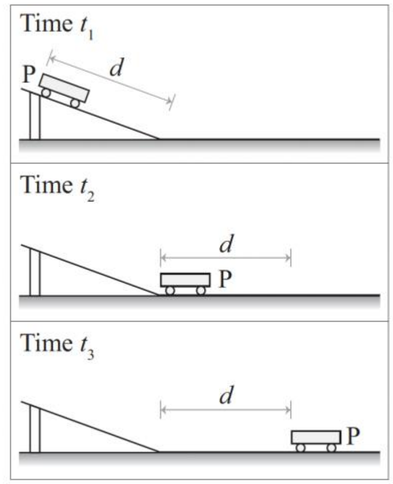
Tutorial Free Response Questions

18. [5 pts] Cart P is released from rest on a frictionless incline at time t_1 as shown. Between times t_1 and t_2, it travels a distance d, reaching the end of the incline. Between times t_2 and t_3, it moves a distance d across a frictionless horizontal surface.



Is $t_3 - t_2$ *greater than, less than,* or *equal to* $t_2 - t_1$? Explain.

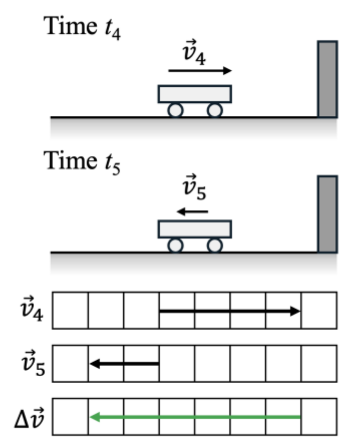
Less than. The time interval $t_2 - t_1$ is equivalent to $d/v_{avg,incline}$. And the time interval $t_3 - t_2$ is equivalent to d/v_{final}, where v_{final} is the cart's speed at the end of the incline.

At t_1, the cart starts from rest and then accelerates to a maximum final speed, v_{final} at t_2. The average speed on the incline must therefore be less than v_{final}.

Since $v_{avg,incline} < v_{final}$, we can conclude that:

$$\frac{d}{v_{final}} < \frac{d}{v_{avg,incline}}, so\ t_3 - t_2 < t_2 - t_1$$

19. [5 pts] At some later time, t_4, the cart has a velocity, $\vec{v}_4$ to the right as shown at right. The cart collides with a wall and its velocity, $\vec{v}_5$, after the collision is also shown. In the space below right, determine the change in velocity of cart from t_4 to t_5. Briefly explain.



The change in velocity between t_4 and t_5 is given as:

$$\Delta\vec{v} = \vec{v}_5 - \vec{v}_4$$

or

$$\Delta\vec{v} + \vec{v}_4 = \vec{v}_5$$

In the latter definition, we can think of the change-in-velocity vector as the vector that is added to $\vec{v}_4$ such that their sum equals $\vec{v}_5$. Since $\vec{v}_4$ is 4 units to the right and $\vec{v}_5$ is 2 units to the left, the change-in-velocity vector must be 6 units to the left.

$$\Delta\vec{v} = (-2\text{ units})\hat{\imath} - (4\text{ units})\hat{\imath} = (-6\text{ units})\hat{\imath}$$

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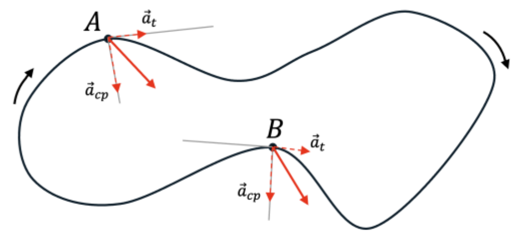
20. [5 pts] An object moves clockwise around the track shown at right (this is a top-view diagram). At point A, the object is speeding up, and at point B the object is slowing down. At each of these points, draw the acceleration vector of the object. Briefly explain.



Top View Diagram

The car is moving clockwise around the track, and at point A it is speeding up. At point A, the acceleration must have a component that is perpendicular to its tangential velocity to account for the change in the car's direction, and it must also have a component in the same direction as its tangential velocity to account for the increase in speed.

At point B, the car is slowing down. The acceleration must have a centripetal component and a tangential component that is in the opposite direction to the tangential velocity.