Q1–Q15: Lecture multiple choice — Indicate your answer on the bubble sheet.

1. [4 pts] An ideal spring-block system is pulled back 3.5 cm from its equilibrium position and then released from rest. It takes 0.09 s for the system to first reach its equilibrium position. What is the frequency of the system?

   A. 2.8 Hz

   B. 3.7 Hz

   C. 5.6 Hz

   D. 8.8 Hz

   E. 11 Hz

   The 0.09-s interval described is only ¼ of a period. Thus the frequency is

   $$f = \frac{1}{4 \times 0.09\ s} = 2.8\ Hz$$

2. [4 pts] A certain simple harmonic oscillator has maximum velocity +5 m/s and maximum acceleration +15 m/s^2. How much time will it take the oscillator to undergo 4 complete oscillations?

   A. 3.5 s

   B. 5.7 s

   C. 8.4 s

   D. 12 s

   E. 14 s

   We can form the ratio $\frac{v_{max}}{a_{max}} = \frac{2\pi f A}{4\pi^2 f^2 A} = \frac{1}{2\pi f} = \frac{T}{2\pi}$ and find that the period is 2.09 s. The requested time interval is four times this amount, or 8.4 s.

3. [4 pts] The frequency of an ideal spring-block system is measured to be 5.8 Hz. You then **double the mass** of the block and pull back the block twice as far before so that its **amplitude is also doubled.** What is the new frequency of the system?

   A. 2.9 Hz

   B. 3.3 Hz

   C. 3.8 Hz

   D. 4.1 Hz

   E. 5.8 Hz

   Amplitude does not affect frequency for simple harmonic oscillators, so we can disregard that information. If the mass m increases by a factor of 2, then the frequency $f = \frac{1}{2\pi}\sqrt{k/m}$ decreases by a factor of $\sqrt{2}$.

4. [4 pts] The angle θ in radians of a certain simple pendulum is described by the equation $\theta(t) = (0.1 \text{ rad}) \sin\left[\left(5.0 \frac{\text{rad}}{\text{s}}\right) t\right]$ where *t* represents time. What is the length of the pendulum?

   A. 0.21 m

   B. 0.25 m

   C. 0.33 m

   D. 0.39 m

   E. 0.45 m

   The constant argument of sine is $2\pi f = 5.0 \frac{\text{rad}}{\text{sec}} = \sqrt{\frac{g}{L}}$. Solving for L yields 0.392 m.

5. [4 pts] Which of the following best describes the properties of visible light and of sound waves in a flute?

A	Visible light: Electromagnetic, longitudinal Sound in a flute: Mechanical, longitudinal
B	Visible light: Electromagnetic, longitudinal Sound in a flute: Mechanical, transverse
C	Visible light: Electromagnetic, transverse Sound in a flute: Mechanical, longitudinal
D	Visible light: Electromagnetic, transverse Sound in a flute: Mechanical, transverse
E	None of these are correct

6. [4 pts] You have a spring of mass 0.04 kg. You fix one end to a wall and exert a horizontal force of magnitude 2.3 N to the other end, which causes the spring to stretch to a length of 3.6 m.

If you quickly shake the end of the spring you are holding, how much time will it take for the resulting pulse to reach the wall?

A. 0.07 s

B. 0.13 s

C. 0.18 s

D. 0.22 s

E. 0.25 s

The time to reach the wall can be computed from $\Delta t = \frac{L}{v} = \frac{L}{\sqrt{\frac{T}{\mu}}} = \frac{\sqrt{L}}{\sqrt{\frac{T}{m}}} = \sqrt{\frac{Lm}{T}}$ where T is tension. Plugging in the three quantities yields 0.250 s.

7. [4 pts] Suppose that the molar mass of a certain diatomic molecule ($\gamma = 1.4$) is 34.0 g/mol = 0.0340 kg/mol. You measure the speed of sound to be 240 m/s in a sample of gas composed entirely of this diatomic molecule at some unknown temperature T_0.

   Later, you replace the diatomic gas with a different diatomic gas at the same unknown temperature T_0. The speed of sound is now measured to be 310 m/s. What is the molar mass of a **single atom** (not a single molecule) of the element that comprises the diatomic molecule?

   A. 56.7 g/mol

   B. 28.4 g/mol

   C. 20.4 g/mol

   D. 14.2 g/mol

   E. 10.2 g/mol

   The speed of sound in a gas is $v = \sqrt{\frac{\gamma RT}{M}}$. We can divide the two given speeds to learn that $\frac{v_2}{v_1} = \sqrt{\frac{M_1}{M_2}}$ which implies the molar mass M_2 of the unknown diatomic gas is $M_2 = M_1\left(\frac{v_1^2}{v_2^2}\right) = 20.38\ \mathrm{g/mol}$. The molar mass of a single atom is half this, or 10.2 g/mol.

8. [4 pts] You and a classmate are holding a very long spring between yourselves. Your friend is shaking the end of the spring and creating sinusoidal traveling waves that **travel at speed v_0** and have **wavelength λ_0.**

   Suppose now that your classmate shakes the spring at a different rate, but no other changes are made to the spring. As a result of this change, the wavelength of the waves is now observed to be $3\lambda_0$. What is the new wave speed of the traveling waves?

   A. $9v_0$

   B. $3v_0$

   C. v_0

   D. $v_0/3$

   E. $v_0/9$

   The medium, which is unchanged, completely determines the wave speed.

9. [4 pts] A traveling transverse wave is described by the equation $y(x,t) = (0.020 \text{ m}) \sin(5.5x + 3.3t)$ where *x* and *t* are measured in meters and seconds, respectively. What are the **speed** and **direction of propagation** of the wave?

A. 0.6 m/s in the +*x* direction

B. 0.6 m/s in the –*x* direction

C. 1.7 m/s in the +*x* direction

D. 1.7 m/s in the –*x* direction

E. None of these are correct.

The signs in front of the *x* and *t* parameters are the same sign, so the wave moves in the negative *x*-direction. The wave speed can be computed from

$$v = \lambda f = \frac{\lambda}{2\pi} 2\pi f = \frac{2\pi f}{\left(\frac{2\pi}{\lambda}\right)} = \frac{3.3}{5.5} \frac{\text{m}}{\text{s}} = 0.6 \text{ m/s}.$$

10. [4 pts] Ultrasonic imaging involves sending ultrasonic waves into the body. The waves leave an emitter at the surface of the skin, reflect from an object within the body, and finally are received by the same instrument that emitted the waves.

An ultrasound technician will study an organ that is 0.0500 m below the surface of the skin. The speed of sound in the body is 1540 m/s. If the technician only wants **one pulse** present in the body at a time, how many pulses per second can the technician send into the body?

A. 15,400

B. 18,000

C. 30,800

D. 36,000

E. 61,600

The total distance traveled by the wave is twice the given distance *d* (there and back), so the number of pulses per second can be computed from $f = \frac{1}{\Delta t} = \frac{v}{2d} = 15{,}400 \text{ 1/s}$.

11. [4 pts] A speaker emits sound with a power rating of 5.0 W. You have a decibel meter and measure the sound intensity level to be 80 dB.

How far away from the speaker are you standing? As usual, assume the speaker emits sound uniformly in all directions. Recall that the intensity corresponding to the threshold of human hearing is 10^{-12} W/m^2.

A. 42 m

B. 63 m

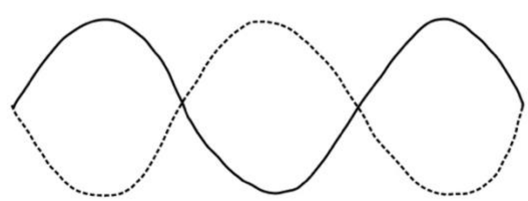
C. 72 m

D. 81 m

E. 95 m

An SIL of 80 dB corresponds to an intensity of $I = I_0 \times 10^{\frac{80}{10}} = 0.0001\ \mathrm{W/m^2}$. We can relate this intensity to power through $I = \frac{P}{4\pi r^2}$. Solving for r yields 63.1 m.

12. [4 pts] The standing wave at right is established in a medium that has length 0.40 m and wave speed 200 m/s. What is the **fundamental frequency** of the medium?



A. 250 Hz

B. 500 Hz

C. 600 Hz

D. 750 Hz

E. 1000 Hz

The fundamental frequency is $f_1 = \frac{v}{2L} = 250$ Hz.

13. [4 pts] Suppose that when a guitar string is plucked, we hear a frequency of 440.0 Hz, which is known as an "A4" note. When the string is pressed just behind the 7^{th} fret of the guitar and plucked, we hear a frequency close to 659.3 Hz, which is an "E5" note. (Recall we discussed frets in class.)

Which of the following **best** explains why we hear a different note when the string is pressed behind the 7^{th} fret?

A. The tension in the guitar string changes by a large amount because the vibrating portion of the string is shorter.

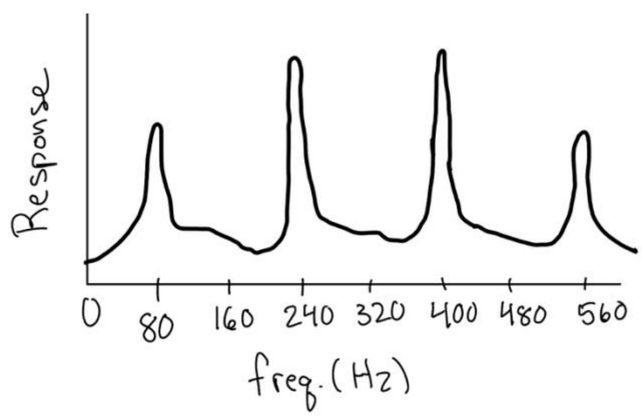
B. The linear mass density in the guitar string changes by a large amount because the vibrating portion of the string is shorter.

C. The fixed-fixed string changes to a fixed-free string because the portion of the string "behind" the fret is now free to oscillate how it wishes.

D. There are now a greater number of antinodes on the vibrating portion of the string, which is now shorter.

E. The fundamental frequency of the string is now different because the vibrating portion of the string is shorter.

14. [4 pts] The diagram at right shows the response curve for a certain instrument that uses standing waves of air (v_{air} = 343 m/s) to produce sound. What is the length of the instrument represented in the diagram?



A. 0.408 m

B. 1.07 m

C. 2.14 m

D. 3.21 m

E. 4.29 m

This is an open-closed instrument since the second, fourth, etc. harmonics are missing. Thus, the length L can be calculated from $f_1 = 80\ Hz = \frac{v_{air}}{4L}$ yielding $L = 1.07$ m.

15. [4 pts] Two different speakers are playing pure sine wave sounds of slightly different frequencies.
    - When **both speakers** are playing, you hear a beat frequency of 4 Hz.
    - When only the **left speaker** is playing, you hear a frequency of 225 Hz.

    What frequency will you hear if only the right speaker is playing?

    A. 4 Hz

    B. 221 Hz

    C. 229 Hz

    D. Could be either B or C

    E. None of the above.

    The beat frequency gives the difference between the two frequencies, but we don't know if the 225 Hz is the higher or lower frequency. Thus, both 221 Hz and 229 Hz are possible.

Q16–Q20: Lecture free response — Unless otherwise noted, explain your reasoning or show your work.

16. [4 pts] A certain simple harmonic oscillator has an equilibrium position at $x = 0$. At time $t = 0$, the oscillator is at the equilibrium position and moving in the negative x-direction. It takes 0.25 s for the oscillator to first return to the equilibrium position. The maximum displacement of the oscillator from equilibrium is 0.40 m.

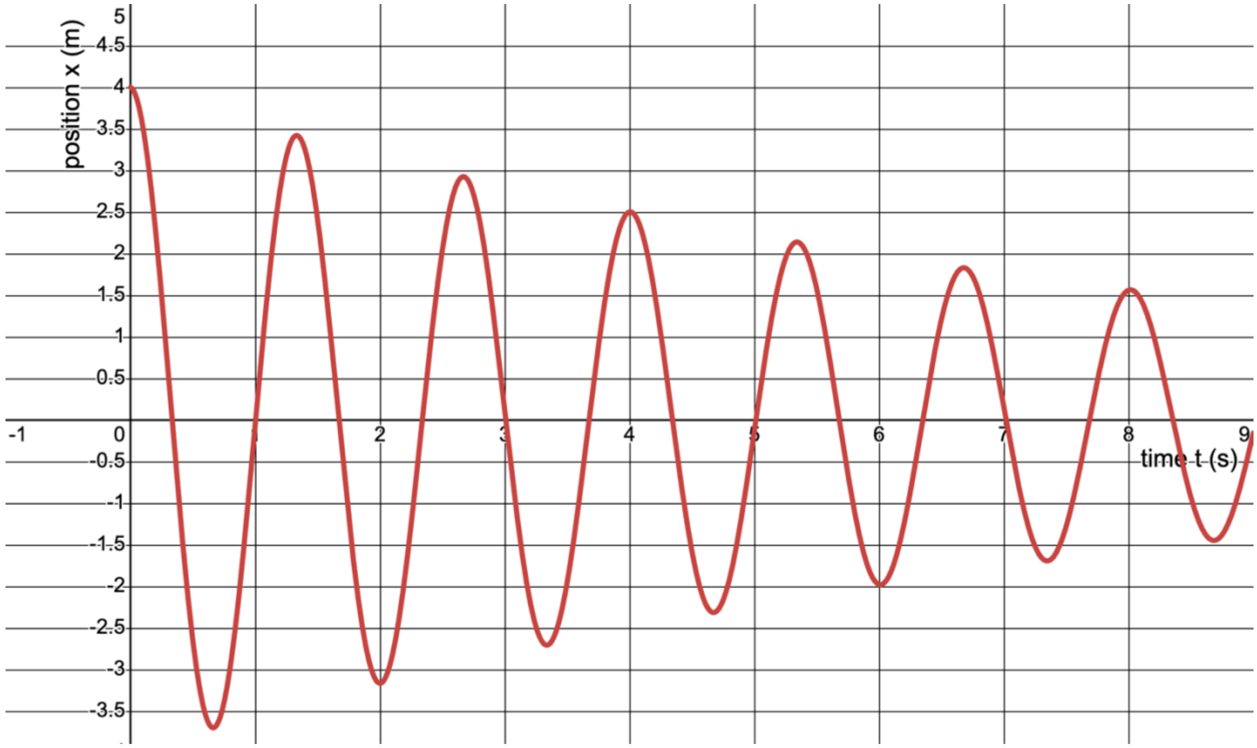
Write down a mathematical function for the velocity $v_x(t)$ of the oscillator. Your function should give the velocity in units of meters/second when time is expressed in seconds. Explain and/or show your work.

The velocity is at its most negative value at $t = 0$, so we must use a negative cosine function: $v_x(t) = -v_{\max}\cos\left(\frac{2\pi t}{T}\right)$. We only need to find the highest speed $v_{\max}$ and the period T.

- The time 0.25 s described in the problem is only *half* of the period since the oscillator completes half of its entire motion, so the period is $T = 0.50$ s which gives $\frac{2\pi}{T} = 12.6\ \mathrm{s}$.
- The highest speed $v_{\max}$ is $A2\pi f = \frac{A2\pi}{T} = \frac{(0.40\ \mathrm{m})2\pi}{0.5\ \mathrm{s}} = 5.03\ \mathrm{m/s}$.

If we let t be in seconds, the velocity in units of meters per second is thus described by $v_{\max} = -5.03\cos(12.6\,t)$.

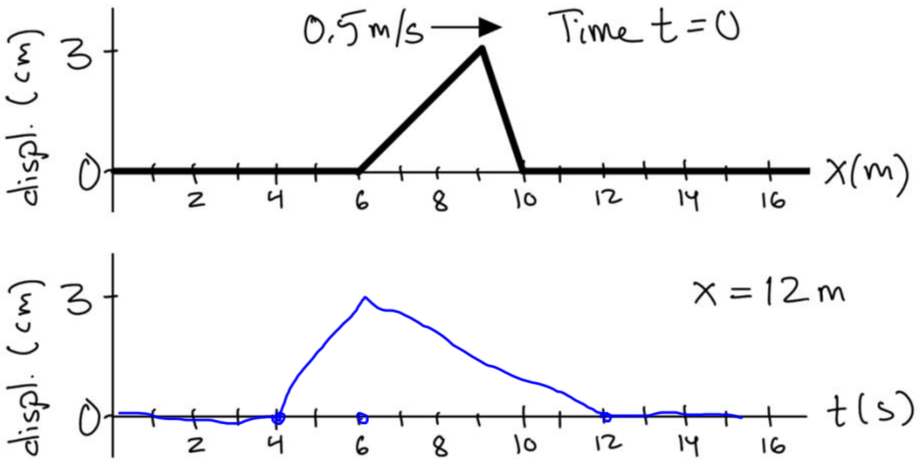
17. [4 pts] The graph below represents position versus time for a damped harmonic oscillator.



At what time will the maximum transverse displacement of the motion be reduced to 0.50 m? Show your work.

At $t = 4$ s we can use $x_{\text{max}}(t) = Ae^{-\frac{t}{\tau}}$ to write $2.5\text{ m} = (4\text{ m})\,e^{\frac{-4\,s}{\tau}}$. Solving for the time constant yields $\tau = \frac{-4\text{ s}}{\ln\left(\frac{2.5}{4}\right)} = 8.51$ s. The time constant can be used to solve for the unknown time by using $0.5\text{ m} = (4\text{ m})\,e^{\frac{-t}{8.51\text{ s}}}$, which yields $t = -(8.51\text{ s})\ln\left(\frac{0.5}{4}\right) =$ 17.7 s.

18. [4 pts] The top diagram represents a snapshot graph of a transverse pulse as it travels in the $+x$-direction at a speed of 0.5 m/s. The snapshot represents the displacement of the spring at time $t = 0$ s.



On the blank axes above, sketch the **history graph** for the **position *x* = 12 m.** No explanation is necessary

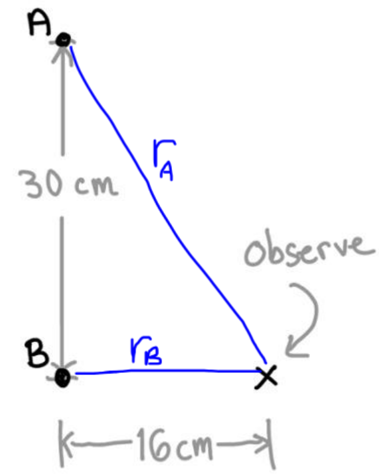
At $t = 0$, the leading edge is 2 m away from $x = 12$ m, so it will take $\Delta t = 2\text{ m}/(0.5\text{ m/s}) =$ **4 s** for this leading edge to begin to affect $x = 12$ m. This line of reasoning can be extended to the peak and to the trailing edge.

19. [4 pts] A fixed-fixed standing wave on a string has 3 antinodes and oscillates at a frequency of 360 Hz. Suppose now that one end of the string is changed to a free end. The frequency is also changed, but the total length, tension, and linear mass density of the string remains the same. As a result of these changes, the standing wave has 4 antinodes.

What is the new frequency? Show your work.

For fixed-fixed, the 3-antinode standing wave has frequency 360 Hz $= f_3 = 3\, f_{\text{fund}}$, so the fundamental frequency is 120 Hz. When changing to a fixed-free, the fundamental frequency is halved—$f_{\text{fund,fixed-fixed}} = v/(2L)$ versus $f_{\text{fund, fixed-free}} = v/(4L)$. This means the new fundamental frequency for fixed-free is 60 Hz. Finally, the allowed frequencies for fixed-free are $f_m = m f_{\text{fund}}$ where $m = 1, 3, 5, 7, 9, \ldots$ so we choose $m = 7$ to get 4 antinodes. Thus, the new frequency is 7(60 Hz) = 420 Hz.

20. [4 pts] Two in-phase identical sources, labeled A and B, are separated by a distance of 30 cm and produce two-dimensional waves. An observation point is located 16 cm to the right of source B.



What is the **largest wavelength** of waves that could produce **complete destructive interference** at the observation point? Assume the amplitude of both waves at the observation point are equal. Show your work.

From the diagram, we can determine

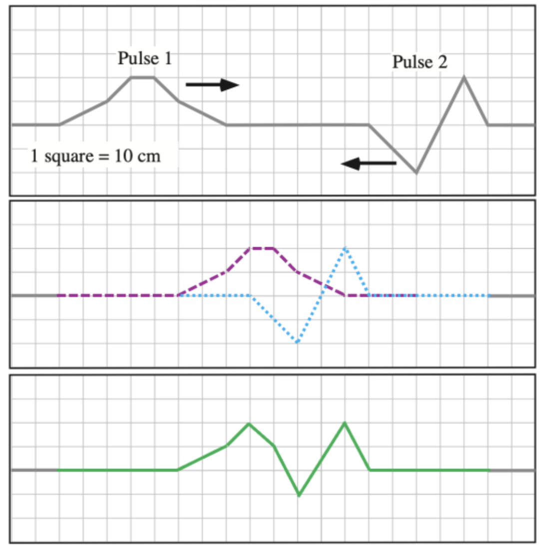
$r_A = \sqrt{(30\text{ cm})^2 + (16\text{ cm})^2} = 34\text{ cm}$ and $r_B = 16\text{ cm},$ so

$\Delta r = 34\text{ cm} - 16\text{ cm} = 18\text{ cm}$. The allowed wavelengths for destructive interference correspond to $\Delta r = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$, so the largest allowed wavelength can be determined using $18\text{ cm} = \frac{\lambda}{2}$ which implies $\lambda = 36\text{ cm}$.

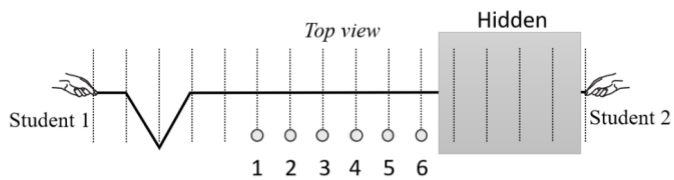
Q21–Q25: Tutorial free response — Unless otherwise noted, explain your reasoning or show your work.

21. [4 pts] Two pulses move towards each other with speed 10 cm/s, as shown below. Each square represents 10 cm.

    On the diagram below right, draw the shape of the spring at the instant $t = 5$ s. No explanation is required.



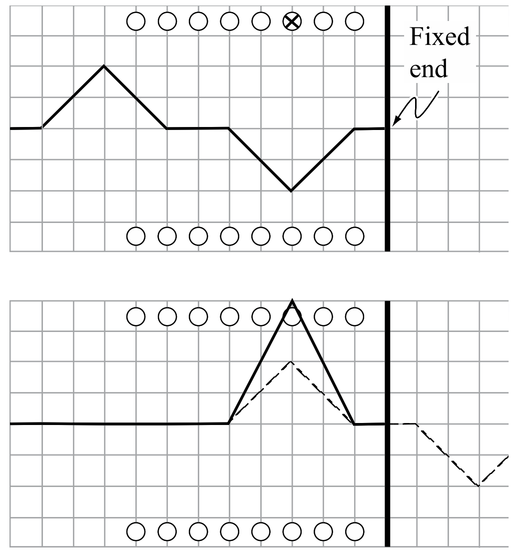
22. [4 pts] Two students hold each end of a stretched spring. Below the stretched spring are six cups. Both students generate a pulse similar in shape, but the pulses are not necessarily generated at the same time. The shape of the spring before the pulses have reached each other, is shown at right (note that any pulse student 2 may have generated is hidden in the image).



    If all cups except cup 5 are knocked over, was student 2's pulse generated *before*, *after*, or *the same time* as student 1's? If this is not possible or if more information is necessary, state so explicitly. Explain your reasoning.

    Cup 5 is located 9 spaces as indicated by the dashed lines and cup is located 6 spaces from student 2. Since the pulses are generated on the same spring, the pulses will move at the same speed. For the pulses to meet at cup 5, the pulse from student 2 would need to be generated after the pulse from student 2, since the pulse from student 1 has farther to travel (and at the same speed).

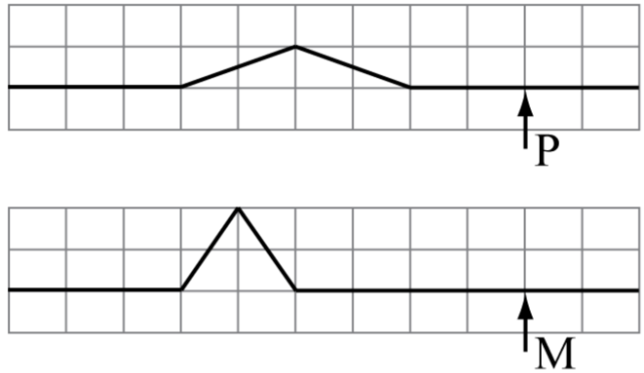
23. [4 pts] A pulse moves to the right along a spring with a fixed end. Cups are placed on either side of the spring as shown. The maximum transverse displacement of the spring while the pulse approaches the end of the spring is less than the distance from the equilibrium position of the spring to the cups. At the time shown, no part of the pulse has reached the fixed end of the spring.



On the diagram at right, mark which (if any) cups are knocked over during the reflection of the pulse from the fixed end. Briefly explain. (You may use the empty grid at right to help you determine your answer, and as part of your explanation, if desired.)

The end of the spring is fixed, so the reflected pulse is on the opposite side of the spring as the incident pulse. The leading peak of the reflected pulse will meet the trailing peak of the incident pulse on the left side of the boundary (as shown at right), knocking over the 3rd cup from the boundary in the top row. No cups from the bottom row will be knocked over; the peaks on the bottom side of the spring meet on the right side of the boundary

24. [4 pts] Two individual pulses are created in springs 1 and 2. Student A measures the time Δt_A for the pulse in spring 1 to pass point P. (The interval of time begins when the leading edge of the pulse reaches point P and ends when the trailing edge passes point P.) Student B measures the time Δt_B for the pulse in spring 2 to pass point M (using the same criteria).



If $\Delta t_B = 2\Delta t_A$, is the speed of the pulse in spring 1 *greater than, less than*, or *equal to* the speed of the pulse in spring 2? Explain.

Greater than. *The width of a pulse is given as,* $width = v_{pulse}\Delta t$, *where* Δt *is the time needed to create the pulse. The time between the leading edge and trailing edge of a pulse passing a particular point is also equal to* Δt.

The width of the pulse in spring 1 (we will call this pulse A) is twice the width of the pulse in spring 2 (pulse B), and the question states $\Delta t_B = 2\Delta t_A$. *We can write the following equations:*

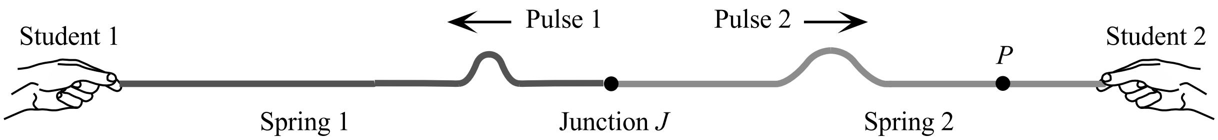
$$w_A = 2w_B$$

$$v_A\Delta t_A = 2v_B\Delta t_B$$

$$v_A\Delta t_A = 2v_B(2\Delta t_A)$$

$$v_A = 4v_B$$

25. [4 pts] Two different springs are connected at a junction *J*. Student 1 holds the left end of spring 1 and student 2 holds the right end of spring 2. Only one student creates a pulse. A short time later, the springs have the shapes as shown in the top view diagram. Ignore reflections at the students' hands.



Top view

Which student generated the incident pulse? If there is not enough information to answer, state so explicitly. Explain.

The reflected pulse is on the same side (both top side) as the transmitted pulse, and the transmitted pulse is always on the same side as the incident pulse. This suggests that the junction behaves more like a free end. Therefore, the incident pulse was generated on the spring with the greater linear mass density μ*. Pulse 1 has traveled a shorter distance from the junction that pulse 2, so the wave speed in spring 1 must be less than that in spring 2. Since speed is inversely proportional to the linear mass density, we can state* $\mu_1 > \mu_2$*. Thus, student 1 generated the incident pulse.*