Name ____ Student ID ____________________

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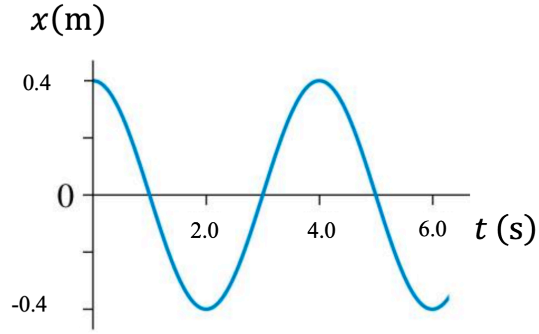
I. Lecture multi choice (60 points)

Use the following scenarios for the following three questions.

The graph below shows the position versus time of a 2kg object undergoing simple harmonic motion.



1) (5 pts.) Which one of the following equations describes the motion of the object?

A. $x(t) = (0.4\mathrm{m})\cos[(1.57\mathrm{s}^{-1})t]$
B. $x(t) = (0.4\mathrm{m})\cos[(0.25\mathrm{s}^{-1})t]$
C. $x(t) = (0.8\mathrm{m})\cos[(1.57\mathrm{s}^{-1})t]$
D. $x(t) = (0.8\mathrm{m})\sin[(0.25\mathrm{s}^{-1})t]$
E. None of the above

- We need to use x(t) = Acos(2πft).
- We don't know f, but we can figure out T from diagram
- T is time from peak to peak, 4s
- Using $f = \frac{1}{T} = \frac{1}{4} = 0.25Hz$
- So, 2πf = 2*3.14*0.24 = 1.57Hz
- From the diagram A = 0.4
- Which gives $x(t) = 0.4\ m \cos\ (1.57\ s^{-1}t)$ (A)

2) (5 pts.) What is the acceleration of the object at time = 3 sec?

A. -2.46 m/s^2
B. -0.99 m/s^2
C. 0 m/s^2
D. 0.99 m/s^2
E. 2.46 m/s^2

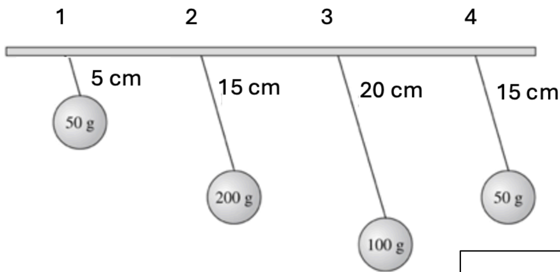
- We need to use $a(t)$ = $-(2\pi f)2A$ cos($2\pi ft$) (eq sheet)
- We can use the same values from problem 1 for A and 2πf
- $a(t)$ = −(1.57)2*0.4 cos(1.57t)
- Finally, plug in t=3
- a(3) = −(1.57)2*0.4 cos(1.57*3) = 0 **(C)**

3) (5 pts.) What is the maximum kinetic energy?

A. 0.25 J
B. 0.39 J
C. 0.63 J
D. 0.79 J
E. 1.57 J

- Kinetic energy is $K = \frac{1}{2}\mathrm{mv}^2$, so we need to find v_{max} to find K_{max}
- We need to use $|V_{max}| = A\omega$ (eq sheet)
- We know A = 0.4 from problem 1, so we must find ω
- $\omega = 2\pi$f (eq sheet)
- So, $|V_{max}| = A\omega = 0.4 * 1.57 = 0.628$
- $K_{max} = \frac{1}{2}mV_{max}^2 = \frac{1}{2}2(0.628)^2 = 0.39J$ **(B)**

4) (5 pts.) Several simple pendulums are shown below, each with varying string lengths and bob masses. All pendulums are initially displaced by the same small angle and then released to oscillate. From the set of pendulums shown, identify pairs where one pendulum oscillates at twice the frequency of the other.



A. Pendulum 1 vs Pendulum 2
B. Pendulum 1 vs Pendulum 3
C. Pendulum 2 vs Pendulum 3
D. Pendulum 2 vs Pendulum 4
E. None of the above

- We need to use: $f = \frac{1}{2\pi}\sqrt{\frac{L}{g}}$ (eq sheet)
- We see f only depends on L and g.
- g doesn't change because they are all in the same place (g is not mass, but gravity constant)
- So only L matters, because of square root if we have 4L we get $2f$
- Which gives use 1 vs 3 **(B)**

5) (5 pts.) A spring-mass system is experiencing damped oscillations. It consists of a 0.5 kg object attached to a spring with a spring constant of 0.5 N/m, and the system has a time constant of 60 seconds. After 20 seconds have elapsed, what fraction of the initial energy remains in the system?

A. 10%
B. 25%
C. 51%
D. 71%
E. 100%

- We need to use: $E = \frac{1}{2}kA^2$ (eq sheet) and E = K + U
- K = 0 at max displacement so, E = U = = $\frac{1}{2}kx_{max}{}^2$
- Using: $x_{max}(t) = Ae^{\frac{-t}{\tau}}$ (eq sheet)
- $x_{max}(0) = A$ and $x_{max}(20) = Ae^{\frac{-20}{60}}$
- $\frac{E_{20}}{E_0} = \frac{\frac{1}{2}k\left(Ae^{\frac{-20}{60}}\right)^2}{\frac{1}{2}kA^2} = \frac{A^2e^{\frac{-20}{60}2}}{A^2} = e^{\frac{-20}{60}2} = e^{\frac{-2}{3}} = 0.51$ **(C)**

6) (5 pts.) When the head vibrates at a frequency that matches the natural frequency of the eyeball in its socket, vision becomes blurred. Consider an eyeball with a typical mass of 7.0×10^{-3} kg, held in place by musculature with an effective spring constant of 210 N/m. Calculate the specific vibration frequency of the head that would cause this blurring effect on vision.

A. 0.87 Hz
B. 5.5 Hz
C. 28 Hz
D. 360 Hz
E. 1088 Hz

- We need to calculate natural frequency of eye
- $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (eq sheet)
- $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{210}{7.0E-3}} = 27.6$ **(C)**

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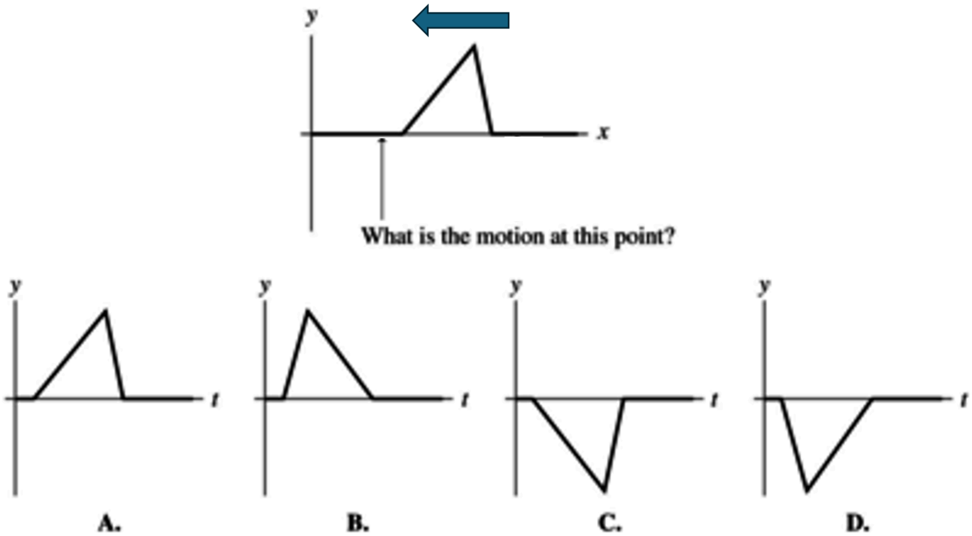
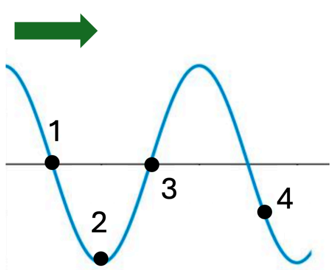
7) (5 pts.) The graph below shows a snapshot graph of a wave on a string that is moving to the **LEFT**. A point on the string is noted. Which of the choices is the correct history graph for the subsequent motion of this point?



A. Graph A
B. Graph B
C. Graph C
D. Graph D
E. None of the above

- We start with a y vs x graph and need to find y vs t graph
- Think about what part of the initial graph touches the point first
- We can eliminate C/D because at no point is the pulse negative
- To select between A/B we must think about what part of the pulse hits the point first. This is the less steep part.
- That means the less steep part will be closer to t=0 on our time graph and the steeper part will be at larger times **(A)**

8) (5 pts.) A snapshot of a wave traveling rightward on a string is shown below. Which of the numbered points on the wave are moving downward in the instant shown?



A. Only point 1
B. Only point 2
C. Only point 3
D. Only point 4
E. More than one of the numbered points.

- Remember transverse waves don't move horizontally, only vertically
- For the wave to move each point, each point will move to the position just left of it in the next instance.
- The positions of the points just to the left of 1,2,4 is above the current point so they will move upward
- The position of the point just to the left of 3 is below it so 3 will move downward. **(C)**

The next two questions will be based on the scenarios described below.

A speaker is producing sound waves that radiate equally in all directions. You are standing 1 meter away from the speaker. The radius of your eardrum is 1×10^{-3} m. At your position, the intensity of the sound reaching your eardrum is measured to be 4.4×10^{-9} watts per square meter (W/m^2).

9) (5 pts.) Determine the total power that your eardrum receives from this sound wave.

A. 1.4×10^{-14} W
B. 5.5×10^{-14} W
C. 3.5×10^{-10} W
D. 1.4×10^{-8} W
E. 5.5×10^{-8} W

- We need to use P= $I \times \pi r^2$ (eq sheet)
- For this case r is the radius of the ear drum, not the distance to the source
- This is because we know the intensity hitting the eardrum, not the intensity that is being emitted.
- P= $I \times \pi r^2 = 4.4 \times 10^{-9}\pi(1 \times 10^{-3})^2 = 1.4 \times 10^{-14} W$ (A)

10) (5 pts.) You then move to a new position 20 meters away from the speaker. Calculate the absolute value of the difference in intensity level (in decibels) between your measurements at these two locations.

A. 13
B. 20
C. 26
D. 40
E. 60

- We need to use $\beta = (10\ dB)\, log_{10}\left(\frac{I}{I_0}\right)$ (eq sheet)
- $I\ = \frac{P}{a} = \frac{P}{4\pi d^2}$ (eq sheet) Tells use the intensity at distance r from the source
- Using this an the properties of logs we can get:
- $\beta_2 - \beta_1 = (10\ dB)\, log_{10}\left(\frac{d_2}{d_1}\right)^2 = 10\, log_{10}(20m/1m)^2 =$ 26 **(C)**

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11) (5 pts.) Consider a sinusoidal wave on a string described by a wave function: $y(x,t) = (0.25\text{m})\cos[(36\text{m}^{-1})x - (25\text{s}^{-1})t]$. Which of the following statements about this wave is/are correct?

I. The wave is traveling in the -x direction

II. The wavelength is 0.17 m

III. The wave speed is 0.69 m/s

- We need to use $y(x,t) = A\cos(2\pi\frac{x}{\lambda} \pm 2\pi\frac{t}{T})$ (eq sheet)
- So $\frac{2\pi}{\lambda} = 36$ and $\frac{2\pi}{T} = 25$
- Thus, $\lambda = 0.17m$ and $f = \frac{1}{T} = \frac{2\pi}{25} = 3.97\ Hz$
- Using: $v = f\lambda$ (eq sheet)
- $v = 3.97 * 0.17$=0.69m/s
- Notice that it is -25t, that means wave is moving to the right (+x) **(D)**

A. I only

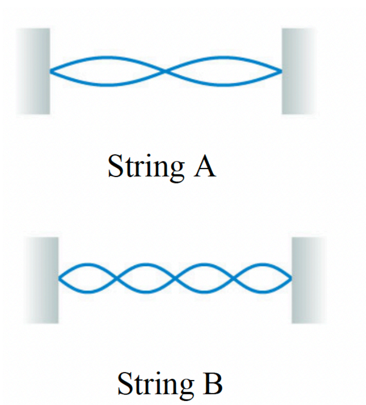
B. II only

C. I and II

D. II and III

E. All statements are correct.

12) (5 pts.) String A is vibrating in a standing wave pattern as depicted below. Identify which modification to the string's conditions would result in the standing wave pattern shown for String B, while maintaining the same oscillation frequency. Note the length of the strings in the figures is not to scale.



- We need to use $f = m\left(\frac{v}{2L}\right)$ (eq sheet)
- Also, Using v $= \sqrt{\frac{T}{\mu}}$
- We get f $= \frac{m}{2L}\sqrt{\frac{T}{\mu}}$
- Taking the ratio of the frequencies, we get $\frac{f_A}{f_B} = \frac{1}{2}\frac{L_B}{L_A}\sqrt{\frac{T_A\mu_B}{T_B\mu_A}}$
- We see that the only valid option is doubling the length with keep the same Tension and Line Density (A)
- We cannot just double/half tension or linear mass density because of the square root. To double the frequency difference, we must change either by a factory of 4.

A. Double the length, keep the same Tension and Line Density

B. Half the Tension, keep the same Length and Line Density

C. Double the Line Density, keep the same Tension and Length

D. Double the Line Density and double the Tension, keep the same Length

E. None of the above

II. Lecture free response (20 points)

13) (6 pts.) A buzzer is being rotated in a circular motion by an instructor, emitting a sound at a constant frequency f_s. The maximum speed at which the buzzer approaches and moves away from you is *V*. When the buzzer is moving at its maximum speed towards you, you hear a frequency f_a. When it's moving away at maximum speed, you hear a frequency f_r. The ratio of these two frequencies (f_a/f_r) is 1.05. Given that the speed of sound is 343 m/s, calculate the maximum speed V of the buzzer's motion.

Ans: When the buzzer approaches and retreats with maximum speed of v_s, you hear frequencies f_a and f_r, respectively.

$$f_a = f_+ = \frac{f_s}{1 - v_s/v}$$
$$f_r = f_- = \frac{f_s}{1 + v_s/v}$$

$$\frac{f_a}{f_r} = \frac{\frac{f_s}{1 - v_s/v}}{\frac{f_s}{1 + v_s/v}} = \frac{v + v_S}{v - v_S} = R \rightarrow v_s = \frac{R-1}{R+1} v = \frac{1.05 - 1}{1.05 + 1} \times 343 = 8.34 m/s$$

14) (6 pts.) Two stationary sound sources are producing beats at a rate of 3 per second when played simultaneously. When source 2 is moved away from source 1 at a constant speed while source 1 remains stationary, the beats cease to be heard by a stationary person. Given that the frequency of source 1 is 600 Hz, determine the frequency of source 2.

Ans:

$$f_{beat} = |f_1 - f_2| = |600\ Hz - f_2| = 3\ Hz$$
$$f_2 = 603\ Hz \text{ or } f_2 = 597\ Hz$$

The source is moving away from the observer, so it experiences a Doppler shift, Therefore the frequency has to be greater than when it is at rest:

$$f_2 = \boxed{603\ Hz}$$

The source is moving toward the observer, so it experiences a Doppler shift, Therefore the frequency has to be less than when it is at rest:

$$f_2 = \boxed{597\ Hz}$$

Students will get full score if argue correctly on either one of the above.

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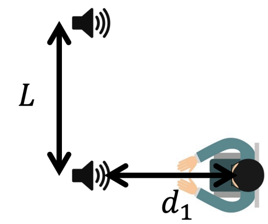
Using the condition below to solve two following questions.

A pair of in-phase stereo speakers is placed side by side, separated by a distance of $L = 2.3$ m. You are sitting a distance $d_1 = 2.06$ m directly in front of one of the speakers. The speakers play a constant note with frequency of 1000 Hz. Assuming the speed of sound is 343 m/s.



15) (4 pts.) How many wavelengths of this sound wave fit in the distance between you and the speaker, d_1?

1) *Ans:* $v = f\lambda$, *so* $\lambda = \frac{v}{f} = \frac{(343\ m/s)}{(1000\ Hz)} = 0.343\ m$

In terms of wavelengths, you are $\frac{d_1}{\lambda} = \frac{(2.06\ m)}{(0.343m)} = 6.00$ *wavelengths in front of the speaker.*

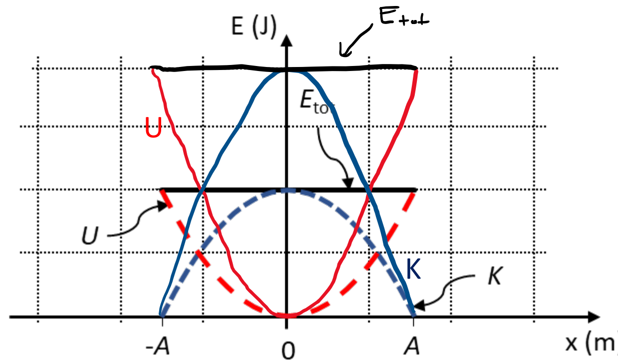
16) (4 pts.) Do you hear constructive or destructive interference?

Ans: We need to determine $\Delta r = d_2 - d_1$ *in terms of the number of* λ*'s.* $d_2 = \sqrt{d_1^2 + L^2} = \sqrt{(2.06\ m)^2 + (2.3\ m)^2} = 3.09\ m$, *or* $\frac{d_2}{\lambda} = \frac{(3.09\ m)}{(0.343\ m)} = 9$ *wavelengths.*

We have determined $d_1 = 6\lambda$. $\Delta r = d_2 - d_1 = 9\lambda - 6\lambda = 3\lambda$ *This results in Constructive interference.*

III. Tutorial free response (20 points)

17) (6 pts.) A block with mass m is attached to a spring with spring constant k. The block is pulled a distance A from the equilibrium position and released such that it oscillates with simple harmonic motion. For the block-spring system the figure shows the potential energy, kinetic energy, and total energy, as a function of the position of the block with respect to its equilibrium position.



Now the spring is replaced with a spring with spring constant $2k$ and the block is again pulled a distance A from the equilibrium position and released.
For the new situation, on the graph above draw and clearly label the following:

A. Potential energy. Explain your reasoning.

We know: $U = 0.5kA^2$ (eq sheet), so if we double k and keep A the same potential energy doubles

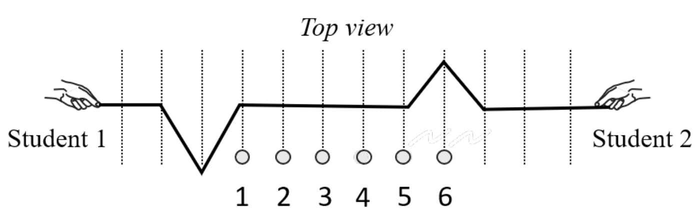
B. Kinetic energy. Explain your reasoning.

E = U + K, because total energy doubled, so does K

C. Total energy. Explain your reasoning.

Energy is conserved throughout the motion so this line is flat. Energy doubles because Potential energy at the maximum doubles

18) (7 pts) Students 1 and 2 each generate a single transverse pulse by moving the ends of a spring at *unknown* times. Originally, there was a line of cups, numbered 1 to 6, below the spring, as shown below. In the time for the pulses to travel to the end of the spring, which of the cups are knocked over? Explain your reasoning.



Cups 1, 2, 4, 5, and 6 are knocked over.
The pulse generated by student 1 extends beyond the cups, so it will knock over the cups. However, the pulse generated by student 2 extends in the direction away from the cups, so when it interferes with student 1's pulse it will create destructive interference, so the waveform will not be large enough to knock over the cup. The pulses from both students are traveling in opposite directions in the same medium, so they have the same speed. Therefore, they will interfere when they have traveled the same distance, so when they are at cup 3.

MORE QUESTIONS ON OTHER SIDE

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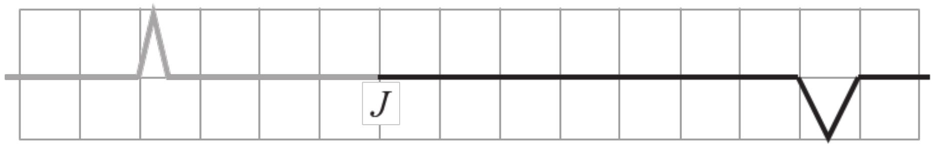
19) (7 pts) Two springs are connected at a junction J. You are not told which spring has a greater mass per unit length. A student generates a pulse at one end of the connected springs. The shape of the springs a short time after the pulse reached junction J is shown below.



Is the mass per unit length of the left spring *greater than*, *less than*, or *equal to*, the mass per unit length of the right spring? Explain your reasoning.

- The pulse on the left is moving slower
  - Pulse is narrower
  - Pulse is closer to junction
- Pulse moving slower means linear mass density is larger
  - Tension in a two-spring system is equal for both springs
  - $v = \sqrt{\frac{T}{\mu}}$, if v is small and T stays the same, μ must be larger