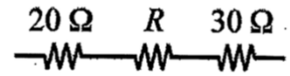
Overall average: 69%

1. [5 pts] Consider the network of three resistors shown at right. If the equivalent resistance of the network of three resistors is 90 Ω, what is the unknown resistance *R*?



98%

A. 10 Ω

B. 20 Ω

C. 30 Ω

D. 40 Ω

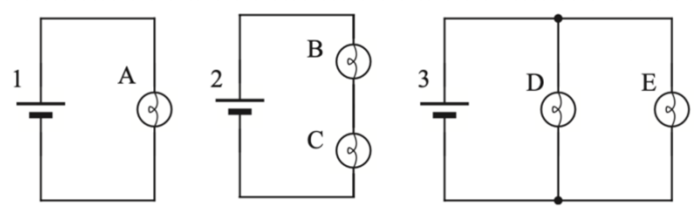
E. 50 Ω

$R_{eq} = 90\,\Omega$ in series $\Rightarrow R_{eq} = 20\,\Omega + R + 30\,\Omega$

$\Rightarrow 90\,\Omega = 20\,\Omega + R + 30\,\Omega$

$\Rightarrow R = 90\,\Omega - 50\,\Omega = 40\,\Omega$

2. [5 pts] Consider the three circuits at right. All batteries are ideal and identical, and all bulbs are identical. Rank the current through the three batteries from largest to smallest.



60%

A. 1 > 2 = 3

B. 1 = 2 = 3

C. 3 > 1 > 2

D. 2 = 3 > 1

E. 2 > 1 > 3

circuit 1: $I_1 = \frac{V}{R}$

circuit 2: $I_2 = \frac{V}{2R}$

circuit 3: $I_3 = \frac{V}{\left(\frac{1}{R}+\frac{1}{R}\right)^{-1}} = \frac{V}{\frac{R}{2}} = \frac{2V}{R}$

$\Rightarrow I_3 > I_1 > I_2$

3. [5 pts] In an experiment, a 12 V battery is connected in series with a $6.0 \times 10^3\,\Omega$ resistor and a 4.0×10^{-5}F capacitor. The capacitor starts uncharged. What is charge on the positive plate of the capacitor after 1.0×10^{-2}s?

64%

A. 1.2×10^{-5} C

B. 2.0×10^{-5} C

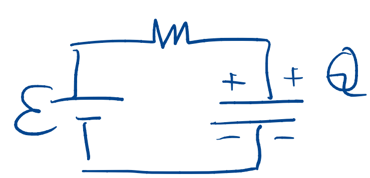
C. 3.5×10^{-5} C

D. 4.8×10^{-5} C

E. 5.8×10^{-5} C

$Q = C\Delta V_c$

$\Delta V_c = \mathcal{E}(1 - e^{-t/\tau})$



$\Rightarrow Q = C\mathcal{E}(1 - e^{-t/\tau})$

$= (4.0 \times 10^{-5}\,F)(12V)\left(1 - e^{-\frac{1.0\times10^{-2}s}{(6.0\times10^3\,\Omega)(4.0\times10^{-5}F)}}\right)$

$\tau = RC$

$= 2.0 \times 10^{-5}\,C$

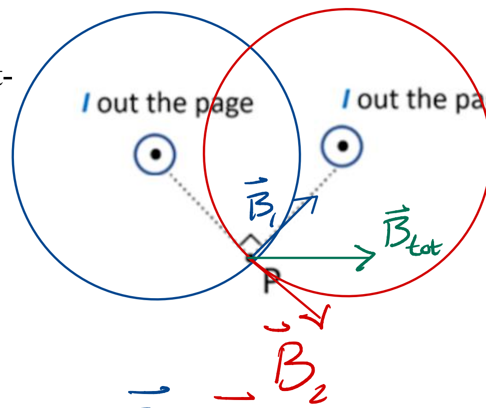
4. [5 pts] Now suppose you repeat the experiment in the previous question, but this time you double the resistance while keeping the battery voltage and capacitor unchanged. You again measure the capacitor's charge at 1.0×10^{-2}s after the circuit is connected (again starting with an uncharged capacitor). Is the charge on the positive plate now *greater than*, *less than* or *equal to* the charge on the positive plate in the experiment in the previous question?
   A. Greater than
   B. Less than
   C. Equal to
   D. Information provided is not enough to answer.

79%

Increasing resistance increases τ $\Rightarrow$ it now takes more time to achieve the same charge.

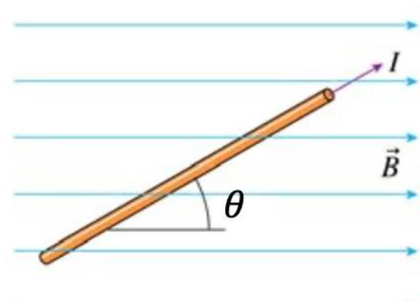
5. [5 pts] Point P is an equal distance from two current-carrying wires and forms the right angle indicated. In what direction is the magnetic field at point P?
   A. Into the page
   B. To the left side of the page
   C. To the right side of the page
   D. Toward the top of the page
   E. Toward the bottom of the page

62%



$\vec{B}_{tot} = \vec{B}_1 + \vec{B}_2$

6. [5 pts] A uniform 2.5 T magnetic field points to the right. A 3.0-m portion of a long wire, carrying current 15 A, is placed at an angle θ to the field, as shown. If the wire experiences a force of 56 N, what is the angle θ?
   A. 35°
   B. 30°
   C. 60°
   D. 25°
   E. 41°



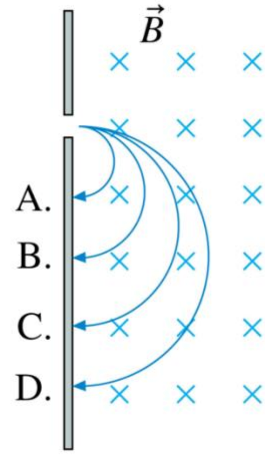
88%

$F = ILB\sin\theta$

$\Rightarrow \sin\theta = \frac{F}{ILB} = \frac{56\ N}{(15A)(3.0m)(2.5T)} = \frac{1}{2}$

$\Rightarrow \theta = \sin^{-1}\frac{1}{2} = 30°$

7. [5 pts] The figure at right shows four particles moving to the right as they enter a region of uniform magnetic field, directed into the paper as shown. All particles have the same speed and the same mass. Which particle has the largest magnitude of charge?



   A. A
   B. B
   C. C
   D. D
   E. They all have the same magnitude of charge.

89%

$qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$

v same, m same, B same

$\Rightarrow r \propto \frac{1}{q} \Rightarrow$ A has smallest r $\Rightarrow$ largest q

8. [5 pts] Wind turbine A can generate electric power from wind at an efficiency of 35%, while wind turbine B's efficiency is 45%. On a certain day, turbine A was able to generate 4.3×10^6 W of electric power, how much electric power did turbine B generate if both turbines are at the same location? $\Rightarrow pay_A = pay_B$

   A. 3.3×10^6 W
   B. 4.3×10^6 W
   C. 8.0×10^6 W
   D. 6.8×10^6 W
   E. 5.5×10^6 W

80%

$e_A = 0.35$ $\quad e_B = 0.45$ $\quad e = \frac{get}{pay}$

$get_A = 4.3 \times 10^6 W$

$\Rightarrow pay = \frac{get}{e} \Rightarrow \frac{get_A}{e_A} = \frac{get_B}{e_B} \Rightarrow get_B = \frac{e_B\, get_A}{e_A}$

$= \frac{0.45}{0.35} 4.3 \times 10^6 W = 5.5 \times 10^6 W$

9. [5 pts] You wish to run a distance of 6.0 km at 15 km/h. Assuming your metabolic power of running at 15 km/h is 1150 W, and if your energy comes from food, which of the following food choices is closest to the minimum amount of energy needed to make the trip?

   A. A slice of pizza supplying 1260 kJ of energy
   B. A frozen burrito supplying 1470 kJ of energy
   C. A slice of apple pie supplying 1680 kJ of energy
   D. A fast-food meal supplying 5660 kJ of energy

83%

$P_{metabolic} = 1150 W = \frac{Q}{\Delta t}$

$v = \frac{\Delta x}{\Delta t} \Rightarrow \Delta t = \frac{\Delta x}{v} = \frac{6.0 km}{15 km/h} = 0.4 hr$

$\Rightarrow Q = (1150 W)(0.4 hr)$

$= (1150 W)(0.4 \times 3600 s) = 1.680 \times 10^6 J$

$= 1680 kJ$

$W_{on\,gas} = +450\,J$

10. [5 pts] A certain amount of gas is inside a cylinder covered by a movable piston. The piston is pushed down slowly doing 450 J of work on the gas. The internal (thermal) energy of the gas is found to have decreased at the end of the process by 320 J. Which one of the following must be true?

$\Delta E_{th} = -320\,J$

A. The gas has lost 770 J of heat to the environment.

B. The gas has absorbed 770 J of heat from the environment.

C. The gas has lost 130 J of heat from the environment.

D. The gas has absorbed 130 J of heat from the environment.

E. Not enough information

$\Delta E_{th} = Q + W_{on\,gas} \Rightarrow Q = \Delta E_{th} - W_{on\,gas} = -320\,J - 450\,J = -770\,J$

11. [5 pts] You toss five identical fair coins (the probability of getting heads equals that of getting tails). What is the ratio of the probability of getting 3 heads to the probability of 4 heads?

A. 1/5

B. 0.5

C. 3/4

D. 4/3

E. 2

$\frac{P(3H)}{P(4H)} = \frac{\Omega(3)/\Sigma\Omega}{\Omega(4)/\Sigma\Omega} = \frac{\Omega(3)}{\Omega(4)}; \quad \Omega(x) = \frac{n!}{x!(n-x)!}$

$\Rightarrow \frac{P(3H)}{P(4H)} = \frac{5!}{3!(5-3)!} \cdot \frac{4!(5-4)!}{5!} = \frac{(4\times 3!)(1!)}{3!(2!)} = 2$

12. [5 pts] An insulated box is divided into two parts by a partition. On the left side, we place a gas at temperature T_L, and on the right side, another gas at temperature T_R, where $T_L < T_R$. We remove the partition and allow the gases to mix. After a long time, what happens to the entropy S of the combined system?

A. It reaches a value S_{avg} representing the average of the initial entropies of the two separate gases.

B. It increases and reaches a maximum value after a while.

C. It keeps increasing with time (with no upper bound).

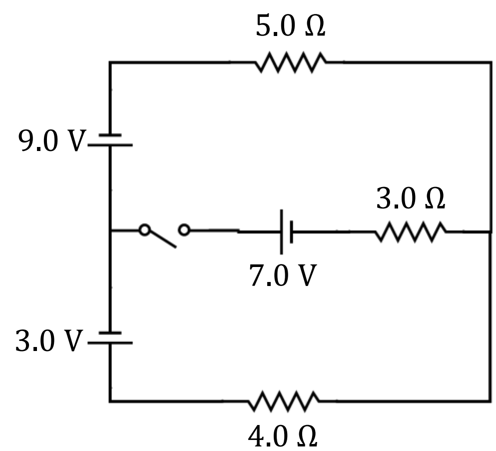
D. It decreases below the entropy of either one of the initial entropies of the two separate gases.

E. It stays the same as before the mixing, since the system is insulated.

Lecture Free response (20 points)

Use the circuit at right to answer the following questions:

13. [4 pts] When the switch is open, how does the current through the 4.0 Ω resistor compare to the current through the 5.0 Ω resistor?



Solution:

When the switch is open, no current flows through the middle branch, which means there will be a single current circulating through the outer loop. This means that the current through the 4.0 Ω resistor is the same as that through the 5.0 Ω one.

60%

Rubric:

4 pts Correct answer (equal to)

0 pts Incorrect/blank

14. [6 pts] With the switch still open, write down a loop equation for the outer loop and use it to find the current flowing through the 4.0 Ω resistor.

Solution:

Since the switch is open and no current flows through the middle branch, we will have one current in the outer loop. Since the two batteries in that loop, $9.0\ V$ *and* $3.0\ V$ *have the same orientation and current flows from high potential to low potential, the current would circulate around the loop counterclockwise. Let point A be near the negative terminal of the* $9.0\ V$ *battery. Going through the loop counterclockwise starting from A and back to A we get:*

69%

$$\Delta V_{AA} = 0 \Rightarrow \boxed{9.0\ V + 3.0\ V - I \cdot (4.0\ \Omega) - I \cdot (5.0\ \Omega) = 0}$$

Now we solve for the equation for current:

$$12\ V = I \cdot (9.0\ \Omega) \Rightarrow I = \boxed{1.3\ A}$$

Rubric:

2 pts for writing a reasonable loop equation for the outer loop (or find an R_{eq} and an equivalent battery). Or 1 pt for a loop equation with error (or attempt to find V_{eq} and R_{eq}).

2 pts Signs of terms in loop equation: 2 pt all signs correct, 1 pt some signs correct 0 pt no signs correct

2 pts for solving for current (1.3 A)

0 Blank or incorrect

15. [6 pts] When the switch is closed, a current of 0.89 A flows right to left through the 3.0 Ω resistor. Write down a loop equation for the upper loop.

Solution:

For the upper loop we have two currents, the one going through the 5.0 *Ω resistor and the* 9.0 *V battery, we'll call it* I_1 *and assume it's going left to right. The other current is the one going through the middle branch, and it's the one given in the question, let's call it* I_2*. To form a loop equation, let's pick the switch S as the reference point, so that:*

57%

$$\Delta V_{SS} = 0$$

Let's travel along the upper loop starting from S and back to it clockwise:

$$\Delta V_{SS} = -9.0\ V - I_1 \cdot (5.0\ \Omega) - I_2 \cdot (3.0\ \Omega) + 7.0\ V = 0$$

Or:

$$2.0\ V + I_1 \cdot (5.0\ \Omega) + I_2 \cdot (3.0\ \Omega) = 0$$

Rubric:

Loop equation:

- 4 pt Write reasonable loop equation with 4 correct terms (9 V, 7 V, 5 Ω, 3Ω)
- 2 pt Semi-correct attempt at writing loop equation.

1 pt Recognize batteries have opposite orientations.

1 pt Recognize current is different in the two resistors.

0 Blank/incorrect

16. [4 pts] Find the value of the current in the 5.0 Ω resistor after the switch has been closed. *Hint*: you can use your answer to the previous question.

Solution:

46%

We are now solving for I_1 *from the equation found in the previous question:*

$$2.0\ V + I_1 \cdot (5.0\ \Omega) + I_2 \cdot (3.0\ \Omega) = 0 \Rightarrow 2.0\ V + I_1 \cdot (5.0\ \Omega) + (0.89\ A) \cdot (3.0\ \Omega) = 0$$

Solving for I_1*:*

$$I_1 = -\frac{2.0\ V + 2.7\ V}{5.0\ \Omega} = \boxed{-0.93\ A}$$

Given that the answer came out negative, it means that the originally assumed direction was wrong, and the current is actually flowing right to left.

Rubric:

2 pts Reasonable algebraic attempt to substitute values of quantities in equation and solve for current.

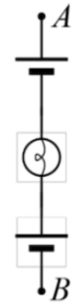
2 pt Correct value of current (0.93A)

0 Blank/incorrect

Tutorial free response (20 points total)

17. Two identical batteries, each with voltage ΔV_{bat}, are connected to a bulb as shown. **No other circuit elements or wires are connected**.

[6 pts] What is the absolute value of the potential difference between points A and B? Explain your reasoning.



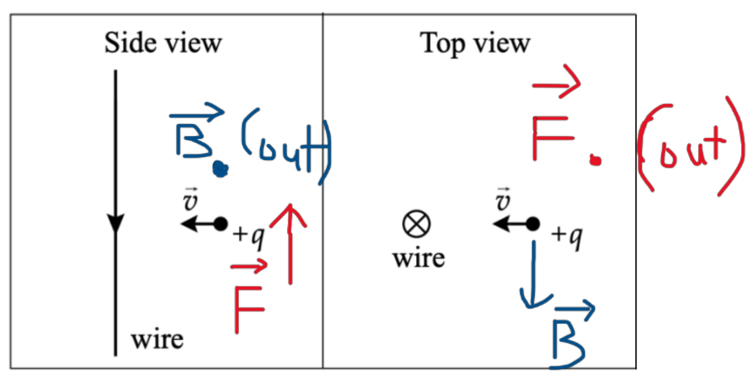
The circuit elements do not form a closed loop, so there is no current, and therefore no potential difference across the bulb. However, the potential difference across each battery is still ΔV_{bat}. Starting from point B and tracking the changes in potential toward point A, we find that

$$|\Delta V_{BA}| = |\Delta V_{bat} + \Delta V_{bulb} + \Delta V_{bat}|$$
$$|\Delta V_{BA}| = |\Delta V_{bat} + 0 + \Delta V_{bat}|$$
$$|\Delta V_{BA}| = 2\Delta V_{bat}$$

47%

Rubric:
2 pts Recognize $I_{bulb} = 0$ and/or $\Delta V_{bulb} = 0$
2 pts Recognize that $\Delta V_{bulb} \neq 0$
2 pts Recognize that $|\Delta V_{BA}| = 2\Delta V_{bat}$
0 Blank/incorrect

18. A very long wire has current directed through it as shown in the top and side view diagrams at right. A point charge $+q$ is moving directly toward the wire with speed v.



65%

[6 pts] In **both the top and side view diagrams**, sketch the direction of the magnetic force on the charged particle. Explain your reasoning.

We can first use the right-hand rule for the direction of the magnetic field from a long straight wire: if our right thumb is the current direction, the field makes circular loops in the direction our fingers curl. The force on the charged particle is then given by the right-hand rule. The force directions are shown in red above.

Rubric:

1 pt Correct $\vec{B}$ direction in **side view**.

1 pt $\vec{F}$ is correct (up the page) or consistent with drawn $\vec{B}$ in **side view**.

1 pt Correct $\vec{B}$ direction in **top view**.

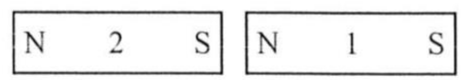
1 pt $\vec{F}$ is correct (out of the page) or consistent with drawn $\vec{B}$ in **top view**.

1 pt Reference right hand rule for force direction.

0 Blank/incorrect

The following two questions are related.

In experiment X, two magnets labeled 1 and 2 are placed next to each other. Magnet 2 is fixed in place.



Experiment X

19. [4 pts] On the diagram at right, indicate the direction of the net force on magnet 1 from magnet 2. If the magnetic force is zero, state so explicitly. Explain your reasoning.

73%

The closest poles on magnet 1 and magnet 2 are opposite in type (one north and one south). There is an attractive force between opposite type poles, so the net force on magnet 1 from magnet 2 is ***to the left****.*

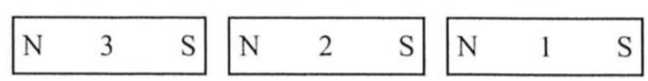
Rubric:

2 pt Recognize there is an attractive force between the nearby opposite poles

2 pt correct direction (to the left)

0 Blank/incorrect

In experiment Y, a third magnet is added to the left of magnet 2 as shown at right. Magnets 2 & 3 are fixed in place.



Experiment Y

20. [4 pts] Is the magnitude of the net force on magnet 1 *greater than*, *less than*, or *the same as* what it was in experiment X above? Explain your reasoning.

76%

For similar reasoning as above, the force on magnet 1 by magnet 3 is to the left as well. The net force on magnet 1 by the other magnets is

$$\vec{F}_{net} = \vec{F}_{2\ on\ 1} + \vec{F}_{3\ on\ 1}$$

which is therefore to the left and ***greater*** *in magnitude than it was in experiment X.*

Rubric:

2 pt correct answer (greater than)

1 pt for discussing the direction of $\vec{F}_{3\ on\ 1}$

1 pt for an argument involving superposition

0 Blank/incorrect