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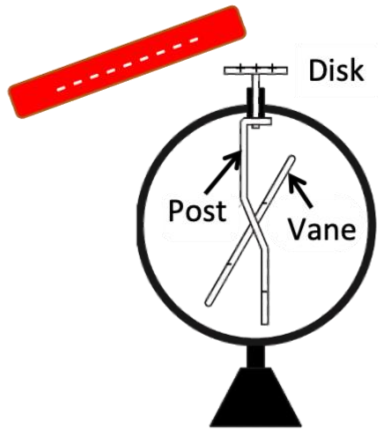
I. [60 pts] Multiple Choice: Mark your answer on BOTH the bubble sheet and this page.

1. [5 pts] The vane on an electroscope is deflected from the vertical direction when a negatively charged insulating rod is brought close to the electroscope (without touching). The following describes different sequences of events.



Case 1: You keep your hand on the disk while the rod is brought close to the electroscope and then move the rod away. You do not move your hand. You can consider yourself as a **conductor.**

Case 2: You keep your hand on the disk while the rod is brought close to the electroscope. You first remove your hand away from the disk, and afterwards you remove the rod.

After the sequences are completed, in which case does the vane remain deflected?

A. Case 1 only

B. Case 2 only

C. Both Case 1 and Case 2

D. Neither Case 1 nor Case 2

E. Not enough information

In Case 1, you become part of the electrically neutral conducting object (electroscope + body) by keeping your hand on the disk. While the rod is close to the electroscope, charge will be polarized in the conductor, making the vane deflect. There will be excess negative charge close to the rod and excess positive charge far from the rod (including your body) due to the repulsive force. After the rod is moved away, the charge in the conductor reaches the electrically neutral equilibrium situation where no charge separation occurs, and the vane will be in a vertical position. In Case 2, the conductor (electroscope + body) will be initially polarized like in Case 1, with significant excess negative charge in the body. When the hand is moved away in this situation (before moving the rod), the negative excess charge remains in the body, leaving the electroscope positively charged. Therefore, even after moving the rod away, there will be a repulsive force between the vane and the post, leaving the vane deflected.

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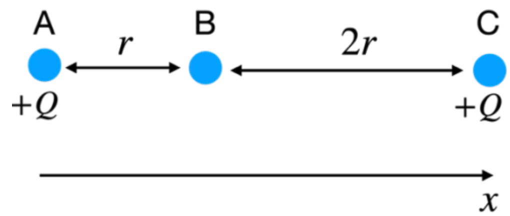
2. [5 pts] Three charged particles are separated as shown in the figure. Particles A and C have equal charge of $+Q$, and the charge of particle B is unknown. Assume that the positive x direction is to the right. The **magnitude** of the force exerted by particle C on particle A is F_0, and the **net force exerted** on particle A is $+2F_0$. What is the charge of particle B?



A. +1/3Q

B. -1/3Q

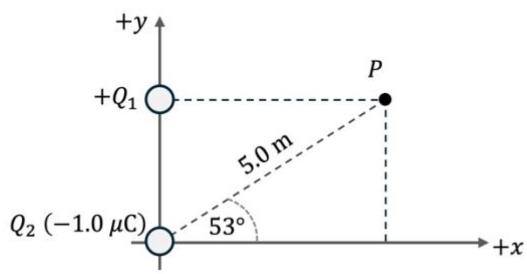
C. +3Q

D. –3Q

E. -Q

The magnitude of the force by particle C on particle A is $F_0 = \frac{KQ^2}{(3r)^2}$ *and its direction is to the left, as the two charges have the same sign, and therefore, the force is repulsive. The net force on charge A is to the right and equal to* $2F_0$*. Therefore, the force from charge B on charge A should be to the right, and it should be equal to* $3F_0$ *to balance out the* F_0 *from C on A to the left. Therefore, B should have a negative charge to exert an attractive force on A. The magnitude of the force follows:* $F_{B\to A} = 3F_0 = K\frac{|Q||Q_B|}{r^2}$*. We know that* $F_0 = \frac{KQ^2}{(3r)^2}$*. Therefore, we arrive at:* $3\frac{KQ^2}{(3r)^2} = \frac{K\,|Q|\,|Q_B|}{r^2}$*. Therefore* $Q_B = \frac{-Q}{3}$*.*

3. [5 pts] Two charges, Q_1 and Q_2 are positioned as shown at right. Q_1 is positively charged, and the magnitude of the electric field at point P due to Q_1 is 560 N/C. What is the magnitude of the **net** electric field at point P?



A. 200 N/C

B. 290 N/C

C. 360 N/C

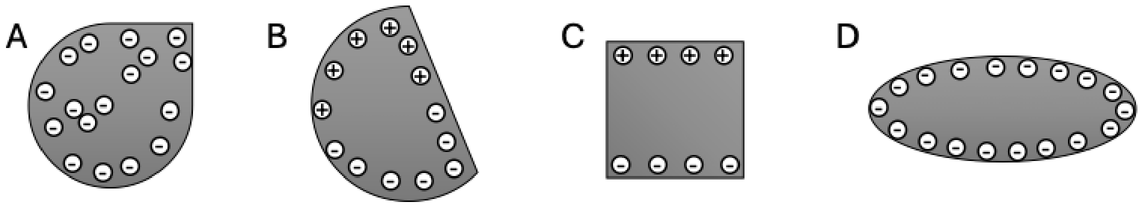
D. 450 N/C

E. 530 N/C

The net electric field at point P is the length of the vector sum of the electric fields of the two point charges. We can infer that the electric field due to $+Q_1$ *points in the positive x direction while the electric field due to* Q_2 *has components in the negative x and negative y directions. Their magnitudes are* $E_{2P,x} = E_{2P}\cos 53^\circ$ *and* $E_{2P,y} = E_{2P}\sin 53^\circ$*, respectively, where* $E_{2P} = \left|\frac{KQ_2}{r_{2P}^2}\right|$ *= 360 N/C with the values given (note that* $r_{2P} = 5.0\ m$*) . Now the magnitude of the net force is*

$E = \sqrt{\left(E_{1P} - E_{2P,x}\right)^2 + E_{2P,y}^2}$ *≈450 N/C.*

4. [5 pts] Which of the figures below correctly illustrates the charge distribution in an ideal conductor of various shapes? There is no external electric field and the conductors are isolated.



A. Figure A

B. Figure B

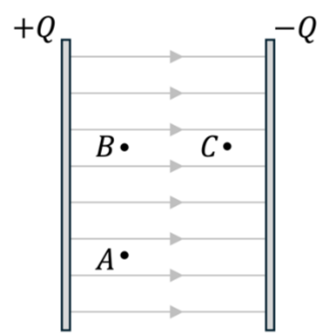
C. Figure C

D. Figure D

E. None of these figures.

For an ideal conductor, excess charges can only be distributed on the outer surface, and only one type of charge can exist. Opposite charges are free to move and recombine.

5. [5 pts] A point charge $+q$ is between the plates of a parallel plate capacitor. Consider moving the point charge first from point A to point B and then from point B to point C. The point charge moves at a constant speed throughout this process. What is true for the external work required to move the $+q$ point charge from A to B ($W_{A\to B}$) and from B to C ($W_{B\to C}$)? Ignore gravitational forces.



A. $W_{A\to B} > 0$ and $W_{B\to C} = 0$

B. $W_{A\to B} < 0$ and $W_{B\to C} < 0$

C. $W_{A\to B} = 0$ and $W_{B\to C} > 0$

D. $W_{A\to B} = 0$ and $W_{B\to C} = 0$

E. $W_{A\to B} = 0$ and $W_{B\to C} < 0$

The line connecting points A and B is perpendicular to the electric field, and therefore no change in the electric potential energy takes place. Also, there is no change in kinetic energy as the charge is continuously moving at constant speed. Therefore, $W_{A\to B} = 0$. *The electric field naturally tends to move the positive point charge from a state of a higher potential energy to a lower potential energy in the direction of the electric field. Due to the conservation of energy, this decrease of potential energy would lead to an increase of kinetic energy. Now, however, the charge is forced to move at constant speed by a force directed opposite to the electric field (to the left), which is opposite to the direction of movement. This leads to negative work, which decreases the overall potential energy of the charge configuration without changing the kinetic energy. Thus* $W_{B\to C} < 0$

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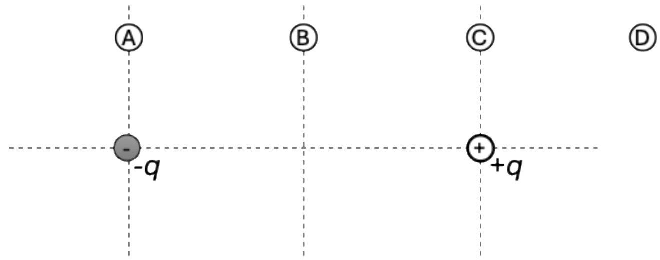
6. [5 pts] Two point particles of charge -*q* and +*q* are placed in space as shown. At which of the points (A, B, C, D) is the total electric potential zero (assuming the potential is zero at infinity)?



   A. Point A

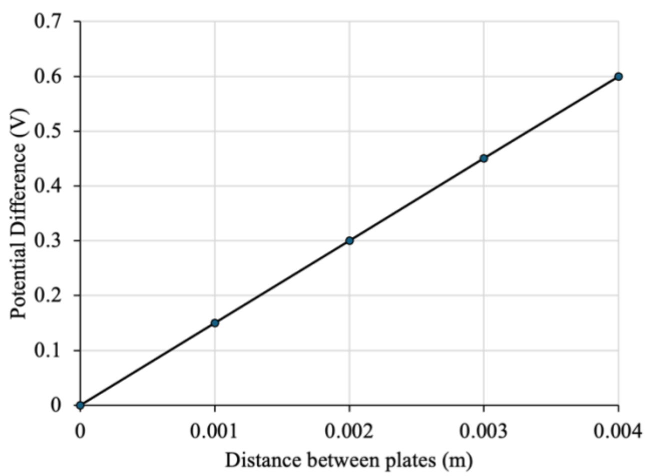
   B. Point B

   C. Point C

   D. Point D

Electrical potential is a scaler (with signs) so can be added. Only point B is at equidistance to the two point charges of opposite signs. Therefore, the electrical potentials of the two charges at point B are equal in magnitude but opposite in sign. So, they add up to be zero. Not the case for point A, C, and D.

7. [5 pts] The graph at right shows how the electric potential difference in a parallel plate capacitor depends on the distance between the plates. What is the magnitude of the electric field between the plates?



   A. 150 V/m

   B. 250 V/m

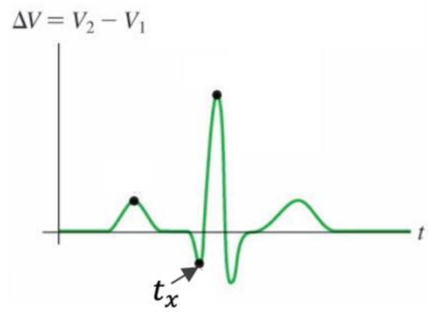
   C. 300 V/m

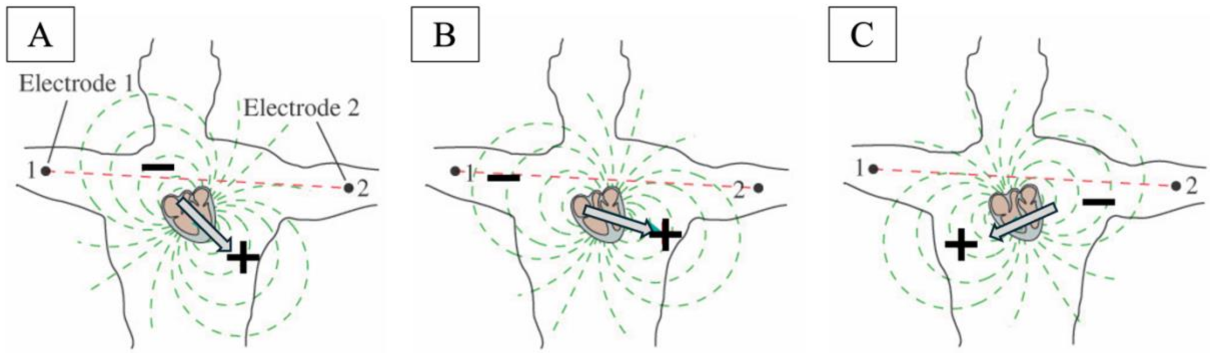
   D. 500 V/m

   E. 1000 V/m

The electric potential difference and the electric field are related as $E = \frac{\Delta V}{d}$*, where d is the distance between the points over which the electric potential difference is being measured. From the given graph, we can read, for example, that the potential difference for a distance of 0.002 m equals 0.3 V. Thus,* $E = \frac{0.3\ V}{0.002\ m} = 150\ V/m$.

8. [5 pts] An electrocardiogram (ECG) produces the potential difference versus time graph shown at right. The ECG measurement is taken by placing two electrodes positioned at points 1 and 2 on the patient, as shown in the figure below. At the time instant, t_x, marked on the graph at right, which diagram (A, B, C) correctly shows the corresponding dipole orientation of the heart, as shown by the arrow in the diagrams below? Note that $\Delta V = V_2 - V_1$.





A. Diagram A

B. Diagram B

C. Diagram C

D. More information is needed.

At time t_x, the ECG measures a negative potential difference of V_2-V_1. It means at that moment, the electric potential at point 1 is higher than point 2. Since the potential difference is generated by the dipole moment of the heart, the heart dipole must be pointing from point 2 toward point 1. Only diagram C shows such a dipole orientation.

9. [5 pts] A parallel plate capacitor consists of two round plates. They are separated by a distance of 0.02 mm and the space between them is filled with a dielectric that has a dielectric constant of $\kappa = 3.2$. What is the radius of the plates if the capacitance of the capacitor is 20×10^{-12} F? Note that the area of a circle is given as: $A_{circle} = \pi r^2$.

A. 2.1 mm

B. 7.7 mm

C. 1.3 mm

D. 0.7 mm

E. 1.1 mm

The capacitance of a parallel plate capacitor with dielectric is $C = \kappa \frac{\epsilon_0 A}{d}$. *Now, we have round plates, so* $A = \pi r^2$, *which leads to* $C = \kappa \frac{\epsilon_0 \pi r^\wedge 2}{d} \Rightarrow r = \sqrt{\frac{Cd}{\kappa \epsilon_0 \pi}}$. *Inserting the given numerical values, we get* $r \approx 2.1\ mm$

10. [5 pts] Charge $Q = 2.4 \times 10^{-3}$C is stored in a capacitor with a capacitance $C = 1.2 \times 10^{-3}$ F. How much electrical energy is stored in the capacitor?

A. 6.0×10^{-4} J

B. 1.2×10^{-3} J

C. 2.4×10^{-3} J

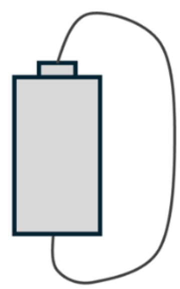
D. 4.8×10^{-3} J

E. 1.6×10^{-2} J

The energy in a capacitor is given as:

$$U = \frac{Q^2}{2C} = \frac{(2.4 \times 10^{-3})^2}{2 \times 1.2 \times 10^{-3}} = 0.0024\ J$$

11. [5 pts] A piece of wire is connected to a battery as shown. For a battery with an emf of 4.0 V, the current in the wire is measured to be 0.15 A. If the wire is ohmic, what is the current in the wire when it is connected to a battery with an emf of 12.0 V?



A. 0.20 A

B. 0.30 A

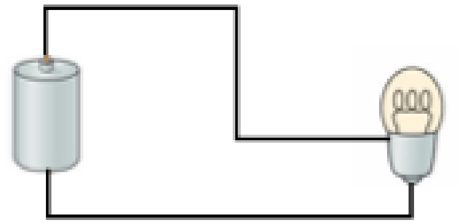
C. 0.45 A

D. 0.68 A

E. 0.10 A

We have an ohmic wire, meaning that the resistance $R = V/I$ *does not depend on the current I. Therefore, the resistance will be the same for an emf of 4.0 V and 12.0 V, and the current* $I = V/R$ *will be proportional to the emf of the battery. So, the current will be 3 times larger for a 12.0 V than for a 4.0 V battery. We know that 4.0 V corresponds to 0.15 A, and therefore the current for the 12.0 V battery will be 3*0.15 A = 0.45 A.*

12. [5 pts] A light bulb is lit up by directly connecting to an AA battery of 1.5 V. Which of the following changes will increase the current through the bulb by a factor of 2?



A. Replacing the battery with a Li-Ion battery of 2 V.

B. Connect another identical light bulb in series with the current bulb.

C. Connect another identical light bulb in parallel with the current bulb.

D. Replacing the battery with two 1.5 V AA batteries connected in parallel.

E. Replacing the battery with two 1.5 V AA batteries connected in series.

From Ohm's law: $I = V/R$, doubling the current requiring doubling the voltage or half the resistance.

Only option E doubles the voltage.

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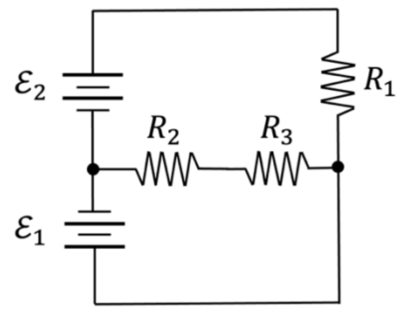
II. Lecture long-answer questions (20 points total)

Consider the following circuit with three resistors for questions 13-14. Assume that the resistors have the following resistances, $R_1 = 2.0\ \Omega$, $R_2 = 4.0\ \Omega$, and $R_3 = 3.0\ \Omega$, and that the EMF of the two batteries are $\mathcal{E}_1 = 20$ V and $\mathcal{E}_2 = 10$ V. In the following questions please show your work to receive full credit.



13. [5 pts] Express the **voltage** across and the **current** through the resistor R_1, first symbolically in terms of the EMF of the two batteries and the resistance, and then numerically. Explain which of Kirchhoff laws you used to arrive at your solution.

We apply the Kirchhoff's loop law, going through the larger loop to calculate the voltage across R_1 *as:* $\mathcal{E}_1 - \Delta V_{R_1} - \mathcal{E}_2 = 0 \rightarrow \Delta V_{R_1} = \mathcal{E}_1 - \mathcal{E}_2 = 10\ V$.

The current can be calculated through Ohm's law: $I_{R_1} = \frac{\Delta V_{R_1}}{R_1} = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1} = \frac{10\ V}{2.0\ \Omega} = 5$ A.

14. [5 pts] What is the magnitude of the currents through R_2 and R_3, respectively? Discuss which Kirchhoff laws you used to arrive at your solution.

R_2 *and* R_3 *are in series and they have the same current through them. The effective resistance of their branch can be replaced by sum of the two* $R_{2+3} = R_3 + R_2$ *. The current through* R_{2+3} *is the same as the current through both* R_2 *and* R_3*. We can use the loop law through the small loop that goes through* $\mathcal{E}_1$ *and* R_1 *and* R_2 *(or equivalently* R_{2+3} *) to compute the voltage across and the current through* R_{2+3}*, which follows:* $\mathcal{E}_1 - \Delta V_{R_2+R_3} = 0 \rightarrow \Delta V_{R_2+R_3} = \mathcal{E}_1$.

Using Ohm's law: $I_{R_2} = I_{R_3} = I_{R_2+R_3} = \frac{\Delta V_{R_2+R_3}}{R_{2+3}} = \frac{\mathcal{E}_1}{R_2+R_3} = \frac{20\ V}{4.0\ \Omega + 3.0\ \Omega} = 2.9\ A$

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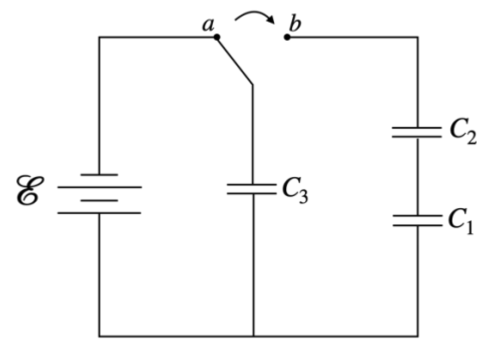
Consider the following circuit with three capacitors for questions 15-16. Initially the switch is at position **a,** and we let the capacitor C_3 to fully charge. The capacitors C_1 and C_2 are uncharged. Then the switch is flipped to position **b.**



Assume that the capacitors have the following capacitances, $C_1 = 40$ μF, $C_2 = 40$ μF, and $C_3 = 20$ μF, and that the EMF of the battery is $\mathcal{E} = 500$ V. In the following questions please show your work to receive full credit.

15. [5 pts] What is the effective capacitance of the circuit after flipping the switch to position **b?** Show your work.

After flipping the switch, capacitor C_3 acts like a battery charging capacitors $C_1$and C_2. In this set up, capacitors $C_1$and C_2 are in series with each other, and they are in parallel with C_3. Therefore, the effective capacitance follows:

$$C_{effective} = C_3 + \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = 20\ \mu\text{F} + \frac{1}{\frac{1}{40\ \mu\text{F}} + \frac{1}{40\ \mu\text{F}}} = 40\ \mu\text{F}$$

16. [5 pts] What are the charge and potential differences across each capacitor after flipping the switch to position **b** and waiting for the circuit to equilibrate? Show your work.

The charge of C_3 before flipping the switch is equal to: $Q_{C_3}^{before} = \mathcal{E}\ C_3 = (500\ V).(20\mu F) = 10^{-2} C$

Since C_1 and C_2 are in series their charges are equal and are equal to the charge of the equivalent capacitor ($C_{1+2} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} = 20\ \mu F$) in that branch.

After flipping the switch, C_3 charges C_1 and C_2 (or the equivalent capacitor in that branch C_{1+2}). *Considering conservation of charge, after flipping the switch, the charge gained by C_{1+2} is equal to the charge lost by C_3. Since $C_3 = C_{1+2}$, the charge on all capacitors will be equal after the switch is flipped. Therefore, C_3 only loses half of its charge to have an equal charge to C_{1+2} after equilibration. Therefore,* $Q_{C_1}^{after} = Q_{C_2}^{after} = Q_{C_3}^{after} = \frac{Q_{C_3}^{before}}{2} = \frac{10^{-2} C}{2} = 0.005$ C

We can then compute the voltage across capacitors:

$$\Delta V_3 = \frac{Q_{C_3}^{after}}{C_3} = \frac{0.005\ \text{C}}{20\ \mu\text{F}} = 250\ \text{V}$$

$$\Delta V_2 = \Delta V_1 = \frac{Q_{C_1}^{after}}{C_1} = \frac{0.005\ \text{C}}{40\ \mu\text{F}} = 125\ \text{V}$$

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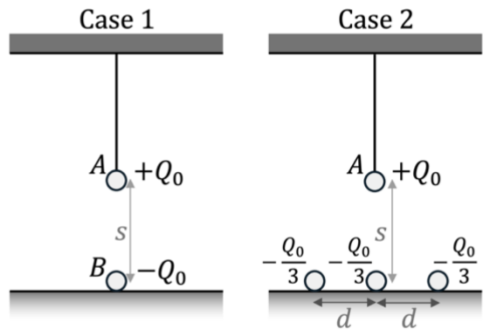
III. Tutorial and lab long answer questions (20 points total)

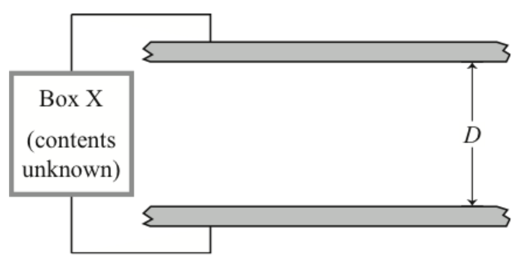
A small sphere, sphere A, with charge $+Q_0$is hung from a light inextensible string. In Case 1 a small sphere, sphere B, with charge $-Q_0$, is fixed on the ground directly below sphere A such that the distance between the centers of the spheres is s. The tension in the string is T_0.



17. [5 pts] In case 2, sphere B from case 1 is replaced with three small spheres, each with charge $-Q_0/3$, and they are fixed in place as shown. Is the magnitude of the tension in the string in case 2 *greater than, less than,* or *equal to* T_0? Explain your reasoning.

    There are three forces exerted on sphere A in case 1, (1) a downward gravitational force (F_g) by the Earth, (2) a downward electric force (F_e) by sphere B, and (3) an upward tension force (T_0) by the string. Since sphere A is at rest, we can write: $T_0 = F_g + F_{e,1}$. In case 2, the small sphere directly below sphere A exerts a force on magnitude $F_e/3$ on sphere A since it has one-third of the charge of sphere B and it located the same distance s from sphere A. The two small spheres to the right and left of the center $-Q_0/3$ charge in case 2 exert a force on sphere A with magnitude less than $F_e/3$, since they are located a distance greater than s and $F_e \propto 1/r^2$. Additionally, in case 2, the x-components of the forces exerted by the left and right charges cancel due to the symmetry in their positions. As a result, the electric force on sphere A in case 2 is less than that in case 1. This means the tension in the string in case 2 is less than T_0.

Two large conducting plates are placed parallel to one another a distance D apart. The plates are connected by wires to box X as shown. When the top plate is moved closer to the bottom plate, it is observed that the electric field between the plates increases.



18. [5 pts] When the top plate is moved closer to the bottom plate, does the absolute value of the charge density on the top plate *increase, decrease,* or *remain the same?* Explain.

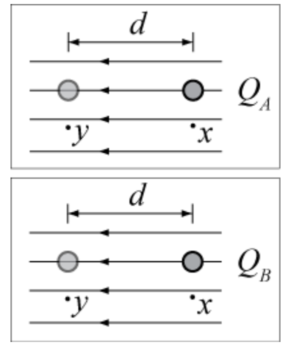
    The magnitude of the electric field between two conducting plates is given as:

$$|\vec{E}| = \frac{Q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0}$$

From this equation, we can conclude that the magnitude of the electric field is only dependent and proportional to the charge density (σ). Since the question states the electric field between the plates increases, the charge density on the plates must also have increased.

Consider this context for questions 19 and 20.

Two particles with the ***same*** positive charge, Q_A and Q_B are released from rest at point *x* in separate uniform electric fields that point in the negative *x*-direction. There is no interaction between the two charges. Both charges move through a distance *d* to the left. (Ignore any gravitational forces.)



19. [5 pts] Is the value of $V_y - V_x$ *positive, negative,* or *zero?* Explain your reasoning.

Electric field and potential analysis

Potential difference can be described as follows:

$$\Delta V = -\vec{E} \cdot \Delta\vec{x}$$

From this equation, we can conclude that the electric potential decreases in the direction of the electric field. Since the electric field points to the left, the potential at point Y must be less than the potential at point X ($V_y < V_x$). This means that the difference, $V_y - V_x$ is negative.

Potential and potential energy analysis

Both charges are positively charged, and experience an electric force to the left, since $\vec{F} = q\vec{E}$. Both charges also move to the left. Since the force on the charge and the charge's displacement point in the same direction, the work done on each charge by the electric field is positive. This will result in an increase in kinetic energy of each charge. Considering a system of the charge and the field, we can account for an increase in kinetic energy by a decrease in electric potential energy of the system.

The electric potential is defined as:

$$\Delta V = \frac{\Delta U}{q}$$

The electric potential energy of the system decreases in both cases, and the charges are positive. The ratio above is therefore negative.

It is known that the mass of Q_A is half as large as the mass of Q_B. When both charges have moved from point *x* to point *y*, the speed of Q_A is measured to be twice the speed of Q_B.

20. [5 pts] Is the magnitude of the electric field strength in which particle A is present *greater than*, *less than*, or *equal to* magnitude of the electric field strength in which particle B is present? Explain your reasoning.

Considering a system that just includes the charge, we can consider the work done on the charge as being equal to its change in kinetic energy. And the work done on the charge is related to the strength of the electric field, as shown below.

$$W = \Delta K$$

$$\vec{F} \cdot \Delta\vec{x} = \Delta K$$

$$q\vec{E} \cdot \Delta\vec{x} = \Delta K$$

Since the electric field and the displacement are parallel in this case, we can write the above equation as follows:

$$qE\Delta x = \Delta K$$

$$E = \frac{\Delta K}{q\Delta x}$$

Let us consider particle A:

$$E_A = \frac{\frac{1}{2}m_A v_{A,f}^2}{q\Delta x}$$

And particle B:

$$E_B = \frac{\frac{1}{2}m_B v_{B,f}^2}{q\Delta x} = \frac{\frac{1}{2}(2m_A)\left(\frac{v_{A,f}}{2}\right)^2}{q\Delta x}$$

$$E_B = \frac{\frac{1}{2}(2m_A)\frac{v_{A,f}^2}{4}}{q\Delta x} = \frac{1}{2}\left(\frac{\frac{1}{2}m_A v_{A,f}^2}{q\Delta x}\right) = \frac{1}{2}E_A$$

From these calculations, we can conclude that $E_A = 2E_B$.