Scale Invariance, Topological Phase Transitions and Invariants
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Instructor: Marcel den Nijs

Topological phenomena, in particular quantum phase transitions with non-local order parameters and TKNN invariant type Berry phases, are at the center of current research. Examples are found in, two dimensional quantum materials, biophysics, quantum information, and non-equilibrium processes. These build directly on research from the 2nd half of the last century. The purpose of this course is to provide graduate students essential background and core materials.

We start with an overview of scale invariance as it emerged around 1960 in experimental work on phase transitions. Scale invariance is the consequence of divergent correlation lengths by strongly fluctuating degrees of freedom. This plays out similarly in most current research. Scale invariance was first observed in experimental data and theoretical series expansions and then explained about 1965 by Kadanoff in terms of "block spin invariance", followed in 1971 by the formulation of renormalization theory (RT) by Wilson and Fisher, while merging with similar ideas and phenomena in particle theory.

Scale invariance is linked to fractal type geometric structures, and RT's can be viewed in retrospect as recursive reformulations of partition functions and correlation functions based on the definition of fractal dimensions, such that exact and approximative methods give the values of the scaling dimensions. We will review RT from this geometric perspective.

Molecular field theory methods date back to van der Waals. They fail fundamentally describing strongly fluctuating collective phenomena and scale invariant systems. These methods are still with us today in, e.g., Landau theory, density functional theory, and effective field theory. We will review the reasons for why they fail.

Next we will discuss examples of topological phase transitions, starting with two dimensional equilibrium critical phenomena, vortices in He films, crystalline surface roughening, and Kosterlitz-Thouless phase transitions. Followed by VBS type phase in quantum spin chains. These phenomena lack local order parameters but display string-like topological order instead. In the current literature on topological insulators it is often claimed that topological phase transitions are fundamentally different from the classic ones with local order parameters, classified by molecular field theories. This is actually not true for most examples I studied in detail. Duality type transformations map topological order into local order. We will review examples of these.

One of the holy grails of current research is to find extensions to higher dimensions of the exact methods that allowed us earlier to determine the exact scaling properties of all one dimensional quantum, and two dimensional classical equilibrium phase transitions. These methods come under several equivalent or complementary names, including, Coulomb gas methods, conformal invariance, and Luttiner-Tomanaga liquid bosonization. We will review aspects of these methods.